(1) $x_i^* \in A_i(x^*)$,

(2) $P_i(x^*) \cap A_i(x^*) = \emptyset$.

THEOREM 1.4.1 : Let $\Gamma = \{(X_i, P_i, A_i) : i \in I\}$ be an abstract economy satisfying the following assumptions for every $i \in I$.

(1) X_i is a nonempty, compact, convex subset of R^ℓ,

(2) P_i has an open graph in $X \times X_i$,

(3) $x_i \notin coP_i(x), \forall x \in X$,

(4) $A_i : X \to 2^{X_i}$ is nonempty, closed, convex valued and continuous correspondence.

Then Γ has an equilibrium, i.e., there exists $x^* \in X$ such that for every $i \in I$,

(a) $x_i^* \in A_i(x^*)$,

(b) $P_i(x^*) \cap A_i(x^*) = \emptyset$

PROOF: For each i, define a correspondence $\varphi_i : X \to 2^{X_i}$ by $\varphi_i(x) := coP_i(x)$. Since P_i has open graph, so does φ_i. For each i, define a correspondence $\psi_i : X \to 2^{X_i}$ by $\psi_i(x) := \varphi_i(x) \cap A_i(x)$. Since φ_i has an open graph and A_i is lower-hemicontinuous, it follows that ψ_i is lower-hemicontinuous. Moreover ψ_i is convex-valued. For each i, define $V_i := \{x \in X : \psi_i(x) \neq \emptyset\}$.

(i) If V_i is empty, (b) is satisfied for all $x \in X$. Since A_i is nonempty, closed, convex valued and continuous, so is A, where $A(x) = \prod_{i \in I} A_i(x)$. By the Kakutani fixed point theorem, there exists $x^* \in A(x^*)$ so that $x_i^* \in A_i(x^*)$. Hence x^* is an equilibrium of Γ.

(ii) Suppose V_i is not empty. Since ψ_i is lower-hemicontinuous, $V_i = \{x \in X : \psi_i(x) \cap X_i \neq \emptyset\}$ is open. Let $\psi_i|_{V_i} : V_i \to 2^{X_i}$ be a restriction of ψ_i to V_i. Then it is nonempty convex valued and lower-hemicontinuous. By the Michael selection theorem, there exists a continuous function $f_i : V_i \to X_i$ such that $f_i(x) \in \psi_i|_{V_i}(x)$ for all $x \in V_i$. For each i, define a correspondence $F_i : X \to 2^{X_i}$ as follows.

$$F_i(x) := \begin{cases} \{f_i(x)\} & \text{if } x \in V_i \\ A_i(x) & \text{if } x \notin V_i \end{cases}$$