(2) $\sum_{i\in I} x_i^* \leq \sum_{i\in I} e_i$

PROOF: Since $\forall p \in \Delta$, $e_i \in \mathcal{B}_i(p)$, $\mathcal{B}_i$ is nonempty-valued. Clearly, $\mathcal{B}_i$ is closed-valued. Since a closed subset of a compact set is compact, $\mathcal{B}_i$ is compact-valued. It is easy to verify that $\mathcal{B}_i$ is convex-valued. Finally, $\mathcal{B}_i$ is continuous. Let $v_i(x,p) := u_i(x)$. By the Maximum Theorem, the demand correspondence φ_i is nonempty-valued, compact-valued and upper hemicontinuous. Furthermore, the quasi-concavity of the utility function and the convex-valuedness of $\mathcal{B}_i$ implies that φ_i is convex-valued. Define the excess demand correspondence $Z : \Delta \to 2^{R^\ell}$ by $Z(p) := \sum_{i\in I} \varphi_i(p) - \sum_i e_i$. Then Z is nonempty compact convex valued and upper-hemicontinuous. Moreover, for every $i \in I$, $\forall p \in \Delta, p \cdot x_i \leq p \cdot e_i$ so that $p \cdot z \leq 0, \forall z \in Z(p)$. By the DGN lemma, $\exists p^* \in \Delta$ such that $Z(p^*) \cap R^\ell_- \neq \emptyset$. Take $z^* \in Z(p^*) \cap R^\ell_-$. Then for every i, there exists $x_i^* \in \varphi_i(p^*)$ such that $\sum_i x_i^* - \sum_i e_i = z^* \leq 0$. Hence (p^*, x^*) constitutes a free disposal equilibrium. □

LEMMA 1.3.2 : If $e_i \in intX_i$, then $\mathcal{B}_i$ is lower hemi-continuous.

1.4 Equilibrium in an Abstract Economy

DEFINITION 1.4.1 : A **game (in a normal form)** $\Gamma = \{(X_i, P_i)_i : i \in I\}$ is a set of pairs (X_i, P_i), where

(1) X_i is the strategy set of player i,

(2) $P_i : X \to 2^{X_i}$ is the preference correspondence player i.

DEFINITION 1.4.2 : $x^* \in X$ is a **Nash equilibrium** if for every $i \in I$, $P_i(x^*) := \{y_i \in X_i : (x_1^*, \ldots, y_i, \ldots, x_n^*) \succ_i (x_1^*, \ldots, x_i^*, \ldots, x_n^*)\} = \emptyset$.

DEFINITION 1.4.3 : An **abstract economy** Γ is a set of triplets $\{(X_i, P_i, A_i) : i \in I\}$ where

(1) X_i is the strategy set of agent i,

(2) $P_i : X \to 2^{X_i}$ is the preference correspondence of agent i,

(3) $A_i : X \to 2^{X_i}$ is the constraint correspondence of agent i.

DEFINITION 1.4.4 : An **equilibrium** for the abstract economy Γ is $x^* \in X$ such that, for every $i \in I$,