By the Brouwer's fixed point theorem, there exists a $p^* = f(p^*) \in F(p^*)$ Hence, $p^* \cdot z > 0, \forall z \in Z(p^*)$. This contradicts the condition (2), which is the Walras law. □

ALTERNATIVE PROOF: (by applying KKM Theorem)
Define $F : \Delta \to 2^{\Delta}$ by $F(p) = \{q \in \Delta : q \cdot z > 0, \forall z \in Z(p)\}$. Then F is convex valued and has open lower sections. Since $F^{-1}(q)$ is open $\forall q \in \Delta$, $G(q) = \Delta \backslash F^{-1}(q)$ is closed $\forall q \in \Delta$. Need to show that $G(q)$ satisfies conditions of KKM theorem.
i) For any set of points $\{q_1, \ldots, q_n\} \subset \Delta$, $co\{q_i, \ldots, q_n\} \subset \bigcup_{i=1}^{n} G(q_i)$.
Suppose not. Let $p \in co\{q_i, \ldots, q_n\}$ and $p \notin \bigcup_{i=1}^{n} G(q_i) \Rightarrow p \notin G(q_i), \forall i \Rightarrow p \in F^{-1}(q_i), \forall i$ $\Rightarrow q_i \in F(p), \forall i \Rightarrow co\{q_i, \ldots, q_n\} \subset coF(p) = F(p) \Rightarrow p \in F(p)$, which is a contradiction to condition (2).
ii) $G(q)$ is compact for each $q \in \Delta$, since it is a closed subset of a compact set Δ.
Therefore, by applying KKM theorem, $\bigcap_{q \in \Delta} G(q) \neq \emptyset$.
Let $p \in \bigcap_{q \in \Delta} G(q) \Rightarrow p \in G(q), \forall q \in \Delta \Rightarrow p \notin F^{-1}(q), \forall q \in \Delta \Rightarrow q \notin F(p), \forall q \in \Delta$ $\Rightarrow F(p) = \emptyset$, for some $p \in \Delta$
But $F(p) = \emptyset$ for some $p \in \Delta$ implies that $\forall q \in \Delta, q \cdot z \leq 0$ for some $z \in Z(p)$, which in turn implies that $Z(p) \cap R^{\ell}_{-} \neq \emptyset$.
To see this, suppose otherwise, i.e., $Z(p) \cap R^{\ell}_{-} = \emptyset$. Since $Z(p)$ is nonempty, compact, convex and R^{ℓ}_{-} is nonempty closed convex, by the separating hyperplane theorem, $\exists q^* \in R^{\ell} \setminus \{0\}$ such that $\sup_{y \in R^{\ell}_{-}} q^* \cdot y < \inf_{z \in Z(p)} q^* \cdot z$. Note that $\sup_{y \in R^{\ell}_{-}} q^* \cdot y = 0$ so that $q^* \cdot z > 0, \forall z \in Z(p)$, a contradiction. Hence for some $p \in \Delta$, $Z(p) \cap R^{\ell}_{-} \neq \emptyset$, as it was to be shown. □

1.3 Existence of Walrasian Equilibrium

THEOREM 1.3.1 : Let $\mathcal{E} = \{(X_i, u_i, e_i) : i \in I\}$ be an exchange economy satisfying the following assumptions for each $i \in I$.

(a) X_i : is a nonempty, compact, convex subset of R^{ℓ},

(b) $u_i : X_i \to R_+$ is quasi-concave and continuous,

(c) $e_i \in intX_i$.

Then $\mathcal{E}$ has a free disposal equilibrium, i.e., there exist $(p^*, x^*) \in \Delta \times X$ with $X = \prod_{i \in I} X_i$ such that

(1) $\forall i \in I, x_i^* \in \varphi_i(p^*) := \{x_i \in \mathcal{B}_i(p^*) : u_i(x_i) \geq u_i(x_i'), \forall x_i' \in \mathcal{B}_i(p^*)\}$, where $\mathcal{B}_i(p^*) := \{x_i \in X_i : p^* \cdot x_i \leq p^* \cdot e_i\}$,