N. B Note that $R(x) = X \setminus P^{-1}(x)$ and $I(x) = R(x) \cap R^{-1}(x)$.

DEFINITION 1.1.2 Define a relation $R : X \to 2^X$ by $R(x) = \{x' \in X : x' \succeq x\}$. The properties of $\succeq$ (or R) are defined as follows. [5]

(1) $\succeq$ is **reflexive** if, for every $x \in X$, $x \succeq x$.

(2) $\succeq$ is **complete** if, for every $x, x' \in X$, $x' \succeq x$ or $x \succeq x'$.

(3) $\succeq$ is **transitive** if $x \succeq x'$ and $x' \succeq x''$ implies that $x \succeq x''$.

(4) $\succeq$ is **weakly monotonic** if $x' \geq x$ implies $x' \succeq x$.

(5) $\succeq$ is **monotonic** if $x' \gg x$ implies $x' \succ x$.

(6) $\succeq$ is **strongly monotonic** if $x' > x$ implies $x' \succ x$.

(7) $\succeq$ is **nonsatiated** if, for every $x \in X$, there is $x' \in X$ such that $x' \succ x$.

(8) $\succeq$ is **locally nonsatiated** if, for every $x \in X$, for every $\varepsilon > 0$, there is a $x' \in B_\varepsilon(x) \cap X$ such that $x' \succ x$.

(9) $\succeq$ is **convex** if $x' \succeq x$ and $x'' \succeq x$ implies $\alpha x' + (1-\alpha)x'' \succeq x$ for every $\alpha \in [0,1]$.

(10) $\succeq$ is **semi-strictly convex** if $x' \succ x$ implies $\alpha x' + (1-\alpha)x \succ x$ for $\alpha \in (0,1]$ and $x' \sim x$ implies $\alpha x' + (1-\alpha)x \succeq x$ for every $\alpha \in [0,1]$.

(11) $\succeq$ is **strictly convex** if $x' \succeq x$ and $x'' \succeq x$ and $x' \neq x''$ implies $\alpha x' + (1-\alpha)x'' \succ x$ for every $\alpha \in (0,1)$.

(12) $\succeq$ has an **extremely desirable bundle** v if for every $x \in X$ and for every $\alpha \in R_+$, $x + \alpha v \in \{x' \in X : x' \succ x\}$.

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[5] Define a relation $P : X \to 2^X$ by $P(x) = X \setminus R^{-1}(x) = \{x' \in X : x' \succ x\}$.

(1) $\succ$ is **irreflexive** if $x \notin P(x)$.

(2) $\succ$ is **transitive** if $x \in P(y)$ and $y \in P(z)$ implies $x \in P(z)$.

(3) $\succ$ is **continuous** if $P(x)$ and $P^{-1}(x)$ are open for every $x \in X$.

(4) $\succ$ is **monotonic** if $x' \gg x$ implies $x' \in P(x)$.

(5) $\succ$ is **strictly monotonic** if $x' > x$ implies $x' \in P(x)$.

(6) $\succ$ is **convex** if P is convex-valued.

(7) $\succ$ is **strictly convex** if $x \neq x'$ implies that $\alpha x + (1-\alpha)x' \in P(x) \cup P(x')$ for every $\alpha \in (0,1)$.