Part II

GENERAL EQUILIBRIUM

1 Walrasian Equilibrium

1.1 Preferences

DEFINITION 1.1.1 A **relation** R is a correspondence from X to 2^X. The properties of R are defined as follows.

(1) R is **reflexive** if, for every $x \in X$, $x \in R(x)$.

(2) R is **irreflexive** if, for every $x \in X$, $x \notin R(x)$.

(3) R is **complete** if, for every $x, x' \in X$, $x' \in R(x)$ or $x \in R(x')$.

(4) R is **transitive** if $x'' \in R(x')$ and $x' \in R(x)$ implies $x'' \in R(x)$.

(5) R is **negatively transitive** if $x'' \notin R(x')$ and $x' \notin R(x)$ implies $x'' \notin R(x)$.

(6) R is **symmetric** if $x' \in R(x)$ implies $x \in R(x')$.

(7) R is **asymmetric** if $x' \in R(x)$ implies $x \notin R(x')$.

(8) R is **antisymmetric** if $x' \in R(x)$ and $x \in R(x')$ implies $x' = x$.

THEOREM 1.1.1 Let $P : X \to 2^X$ be a relation.

(1) If P is asymmetric, then it is irreflexive.

(2) If P is asymmetric and negatively transitive, then it is transitive.

THEOREM 1.1.2 Define relations $R : X \to 2^X$ and $I : X \to 2^X$ by $R(x) := \{x' \in X : x \notin P(x')\}$ and $I(x) = \{x' \in X : x' \in R(x) \text{ and } x \in R(x')\}$. Then

(1) P is asymmetric iff R is complete.

(2) P is negatively transitive iff R is transitive.

(3) P is asymmetric and negatively transitive implies that I is reflexive, symmetric, and transitive.