

(1) We can show that

$$\begin{aligned}
P(x_8) &= \{\{\omega_1, \omega_2, \omega_3\}, \{\omega_4\}, \{\omega_5\}\}, \\
P(x_2, x_3) &= \{\{\omega_1\}, \{\omega_2\}, \{\omega_3\}, \{\omega_4\}, \{\omega_5\}\} = P(x_2) \vee P(x_3), \\
\sigma(x_7) &= \{\{\omega_1, \omega_5\}, \{\omega_2, \omega_3\}, \{\omega_4\}, \{\omega_1, \omega_4, \omega_5\}, \{\omega_2, \omega_3, \omega_4\}, \{\omega_1, \omega_2, \omega_3, \omega_5\}, \emptyset, \Omega\}, \\
\sigma(x_3, x_6) &= \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}, \{\omega_5\}, \{\omega_1, \omega_2, \omega_5\}, \{\omega_3, \omega_4, \omega_5\}, \{\omega_1, \omega_2, \omega_3, \omega_4\}, \emptyset, \Omega\} \\
&= \sigma(x_3) \vee \sigma(x_6).
\end{aligned}$$

(2) x_1 and x_7 are $(P_1 \vee P_2)$ -measurable, x_6 is $(P_1 \wedge P_2)$ -measurable, x_8 is $(P_1 \wedge P_3)$ -measurable and x_5 is P_i -measurable for every i .

(3) Since x_3 is P_2 -measurable and P_3 is finer than P_2 , x_3 is also P_3 -measurable.

(4) Since x_2 is P_1 -measurable, $3x_2 = (0, 3, 3, 12, 6)$ is still P_1 -measurable.

(5) $x_2 - 2x_3 = (-2, -1, -5, -2, -2)$ is $(P_1 \vee P_2)$ -measurable.

(6) $(x_3, x_4) = ((1, 0), (1, 0), (3, 2), (3, 4), (2, 5))$ is $(P_2 \vee P_3)$ -measurable.

(7) $x_2 x_4 = (0, 0, 2, 16, 10)$ is $(P_1 \vee P_3)$ -measurable.