1. The information set of agent 2 at $\omega_3$ is $P_2(\omega_3) = \{\omega_3, \omega_4\}$.

2. An event $A_1 = \{\omega_1, \omega_2, \omega_3\}$ occurs at $\omega_2$.

3. Agent 1 knows that $A_1$ occurs at $\omega_1$, and knows that $P_1(\omega_2) = \{\omega_2, \omega_3\}$ occurs at $\omega_2$.

4. $P_3$ is finer than $P_2$ and $P_2$ is coarser than $P_3$.

5 $\sigma(P_2) = \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}, \{\omega_5\}, \{\omega_1, \omega_2, \omega_3, \omega_4\}, \{\omega_1, \omega_2, \omega_5\}, \{\omega_3, \omega_4, \omega_5\}, \emptyset, \Omega\}$.

6. We can show that

$$\begin{aligned} P_1 \wedge P_2 &= \{\{\omega_1, \omega_2, \omega_3, \omega_4\}, \{\omega_5\}\}, \\ P_1 \wedge P_3 &= \{\{\omega_1, \omega_2, \omega_3\}, \{\omega_4\}, \{\omega_5\}\}, \\ P_2 \wedge P_3 &= P_2. \end{aligned}$$

Note that $P_1 \wedge P_2$ is coarser than $P_1$.

7. We can show that

$$\begin{aligned} P_1 \vee P_2 &= \{\{\omega_1\}, \{\omega_2\}, \{\omega_3\}, \{\omega_4\}, \{\omega_5\}\}, \\ P_1 \vee P_3 &= P_1 \vee P_2, \\ P_2 \vee P_3 &= P_3. \end{aligned}$$

Note that $P_1 \vee P_2$ is finer than $P_1$.

8. We can show that

$$\bigwedge_{i=1}^{3} P_i = P_1 \wedge P_2, \quad \bigvee_{i=1}^{3} P_i = P_1 \vee P_2.$$

9. $A_2 = \{\omega_1, \omega_2, \omega_3, \omega_4\}$ is common knowledge of $\{1, 2, 3\}$ at $\omega_2$ and $A_3 = \{\omega_5\}$ is commom knowledge of $\{1, 2, 3\}$ at $\omega_5$, but $A_1 = \{\omega_1, \omega_2, \omega_3\}$ is not common knowledge of $\{1, 2, 3\}$ at any $\omega \in \Omega$. Note that at $\omega_2$, every agent knows that $A_1$ occurs. However, $A_1$ is common knowledge of $\{1, 3\}$ at $\omega_2$.

10. Let us write $x = (x(\omega_k))_{k=1}^5$. Consider the following random variables $x_1, \ldots, x_8$ :

$$\begin{aligned} x_1 &= (1, 2, 3, 4, 5), \quad x_2 = (0, 1, 1, 4, 2), \\ x_3 &= (1, 1, 3, 3, 2), \quad x_4 = (0, 0, 2, 4, 5), \\ x_5 &= (1, 1, 1, 1, 1), \quad x_6 = (1, 1, 1, 1, 5), \\ x_7 &= (1, 0, 0, 3, 1), \quad x_8 = (0, 0, 0, 1, 2); \end{aligned}$$