DEFINITION 10.3: An event A **occurs at** ω if $\omega \in A$.

DEFINITION 10.4: Agent i **knows that** A **occurs at** ω if $P_i(\omega) \subset A$. **N. B.** For every $\omega \in \Omega$, agent i knows that $P_i(\omega)$ occurs at ω.

DEFINITION 10.5: An event A is **common knowledge of** S **at** ω if $(\bigwedge_{i \in S} P_i)(\omega) \subset A$.

In the game theory context, common knowledge is defined by different, but equivalent way. For more, refer to chapter 5 of Osborne-Rubinstein or chapter 14 of Fudenberg-Tirole.

DEFINITION 10.6: A function $x : \Omega \to R$ is P_i**-measurable** if $x^{-1}(B) \in \sigma(P_i)$ for every $B \in \mathcal{B}(R)$.

DEFINITION 10.7: A partition $P(x_1, \ldots, x_n)$ is the smallest partition with respect to which $x_1, \ldots, x_n$ are measurable. It is said to be **generated by** $x_1, \ldots, x_n$.

THEOREM 10.3 Let x_i be P_i-measurable for every i.

(1) If P_i' is finer than P_i, then x_i is P_i'-measurable.

(2) If $\lambda \in R$, then λx_i is P_i-measurable.

(3) $\sum_{i \in S} x_i$ is $(\bigvee_{i \in S} P_i)$-measurable, where $(\sum_{i \in S} x_i)(\omega) = \sum_{i \in S} x_i(\omega)$.

(4) $(x_i)_{i \in S}$ is $(\bigvee_{i \in S} P_i)$-measurable, where $\left((x_i)_{i \in S}\right)(\omega) = (x_i(\omega))_{i \in S}$.

(5) $\prod_{i \in S} x_i$ is $(\bigvee_{i \in S} P_i)$-measurable, where $(\prod_{i \in S} x_i)(\omega) = \prod_{i \in S} x_i(\omega)$.

Example

Let $\Omega = \{\omega_1, \omega_2, \ldots, \omega_5\}$ and $\mathcal{F} = 2^{\Omega}$. Consider three agents whose information is given by

$$\begin{aligned} P_1(\Omega) &= \{\{\omega_1\}, \{\omega_2, \omega_3\}, \{\omega_4\}, \{\omega_5\}\}, \\ P_2(\Omega) &= \{\{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}, \{\omega_5\}\}, \\ P_3(\Omega) &= \{\{\omega_1, \omega_2\}, \{\omega_3\}, \{\omega_4\}, \{\omega_5\}\} \end{aligned}$$

We will use P_i instead of $P_i(\Omega)$ by abusing of notation.