Lemma An information correspondence P is an information partition if and only if it satisfies (i) and (ii).

The **(private) information** of agent i is represented by an information partition P_i on Ω. The **information set** of agent i at ω is given by $P_i(\omega)$.
Definition :

(1) An information P' is **finer** than an information P and P is **coarser** than P' if $P'(\omega) \subset P(\omega)$, $\forall\, \omega \in \Omega$.

(2) A **meet**, $\bigwedge_{i\in S} P_i$, of $\{P_i : i \in S\}$ is the finest information that is coarser than P_i for every $i \in S$.

(3) A **join**, $\bigvee_{i\in S} P_i$, of $\{P_i : i \in S\}$ is the coarsest information that is finer than P_i for every $i \in S$.

N. B. Note that P' is finer than P iff $\sigma(P) \subset \sigma(P')$ where $\sigma(P)$ is the smallest σ-field containing the partition generated by P.

N. B. The finest information is given by P such that $P(\omega) = \{\omega\}$ for every $\omega \in \Omega$. The coarsest information is given by P such that $P(\omega) = \Omega$ for every ω.

THEOREM 10.1: The following hold :

(1) For every $\omega \in \Omega$, $(\bigwedge_{i\in S} P_i)(\omega) = \bigcup_{i\in S}\{P_i(\omega') : \omega' \in (\bigwedge_{i\in S} P_i)(\omega)\}$,

(2) $\sigma(\bigwedge_{i\in S} P_i) = \bigcap_{i\in S} \sigma(P_i)$,

(3) For every $\omega \in \Omega$, $(\bigvee_{i\in S} P_i)(\omega) = \bigcap_{i\in S}[P_i(\omega)]$,

(4) $\sigma(\bigvee_{i\in S} P_i) = \sigma(\bigcup_{i\in S} P_i)$.

THEOREM 10.2: If P' is finer than P, then $P' \wedge P = P$ and $P' \vee P = P'$.

Given our interpretation of an information correspondence, a decision-maker for whom $P(\omega) \subset A$ knows, in the state ω, that some state in the event A has occurred. In this case we say that in the state ω the decision-maker knows A. For every $\omega \in \Omega$, agent i knows that $P_i(\omega)$ occurs at ω because of property (i).