Lemma (Fatou) : Let $(\Omega, \mathcal{F}, \mu)$ be a finite measure space and (f_n) be a sequence of measurable functions on Ω.

$$\int \liminf f_n d\mu \leq \liminf \int f_n d\mu$$

Dominated Convergence Theorem : Let $(\Omega, \mathcal{F}, \mu)$ be a finite measure space and (f_n) be a sequence of measurable functions on Ω. Suppose that g is a nonnegative integrable function on Ω and f is a measurable function Ω such that

$$|f_n| \leq g, \forall n \in N \text{ and } f_n \to f, \mu\text{-}a.e..$$

Then f and $f_n, n \in \mathcal{N}$ are integrable and $\int f_n d\mu \to \int f d\mu$.

10 Information Structure

We have a **probability measure space** $(\Omega, \mathcal{F}, \mu)$. There are m agents and μ is their **common prior**.

DEFINITION 10.1: An **information correspondence** is a nonempty-valued correspondence $P : \Omega \to \mathcal{F}$.

The interpretation is that when the state is ω the decision-maker knows only that the state is in $P(\omega)$. When we use information correspondence to model a decision-maker's knowledge we usually assume the following two conditions:

(i) For every $\omega \in \Omega$, $\omega \in P(\omega)$,

(ii) If $\omega' \in P(\omega)$, then $P(\omega') = P(\omega)$.

(i) says that the decision maker never excludes the true state from the set of states he regards as feasible. (ii) says that the decision-maker uses the consistency or inconsistency of states with his information to make inferences about the state.

DEFINITION 10.2: An information correspondence P is an **information partition** if there is a partition $\mathcal{P}$ such that for every $\omega \in \Omega$, $\omega \in P(\omega) \in \mathcal{P}$. The partition $\mathcal{P}$ is said to be **generated by** P.

N. B. An information partition P can be identified with the partition that it generates.