(3) $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4, \omega_5\}$.
$\mathcal{F} = \{\{\omega_1\}, \{\omega_5\}, \{\omega_1, \omega_5\}, \{\omega_2, \omega_3, \omega_4\}, \{\omega_1, \omega_2, \omega_3, \omega_4\}, \{\omega_2, \omega_3, \omega_4, \omega_5\}, \emptyset, \Omega\}$ is a σ-field.

DEFINITION 9.5 : Let $\mathcal{F}_1, \mathcal{F}_2$ be σ-fields on Ω.

(1) $\mathcal{F}_1$ is **finer** than $\mathcal{F}_2$ and, $\mathcal{F}_2$ is **coarser** than $\mathcal{F}_1$ if $\mathcal{F}_2 \subset \mathcal{F}_1$.

(2) The **join** $\mathcal{F}_1 \vee \mathcal{F}_2$ of $\mathcal{F}_1$ and $\mathcal{F}_2$ is the smallest σ-field containing both $\mathcal{F}_1$ and $\mathcal{F}_2$.

(3) The **meet** $\mathcal{F}_1 \wedge \mathcal{F}_2$ of $\mathcal{F}_1$ and $\mathcal{F}_2$ is the largest σ-field contained in both $\mathcal{F}_1$ and $\mathcal{F}_2$.

Example :

$$\begin{aligned} \Omega &= \{\omega_1, \omega_2, \omega_3\}, \\ \mathcal{F}_1 &= \{\{\omega_1, \omega_2\}, \{\omega_3\}, \emptyset, \Omega\}, \\ \mathcal{F}_2 &= \{\{\omega_1, \omega_3\}, \{\omega_2\}, \emptyset, \Omega\}. \end{aligned}$$

Then

$$\begin{aligned} \mathcal{F}_1 \vee \mathcal{F}_2 &= \{\{\omega_1\}, \{\omega_2\}, \{\omega_3\}, \{\omega_1, \omega_2\}, \{\omega_1, \omega_3\}, \{\omega_2, \omega_3\}, \emptyset, \Omega\}, \\ \mathcal{F}_1 \wedge \mathcal{F}_2 &= \{\emptyset, \Omega\}. \end{aligned}$$

DEFINITION 9.6: A **finite partition** of Ω is a finite family of disjoint subsets of Ω, whose union is Ω.

DEFINITION 9.7: A partition $\mathcal{F}'$ of Ω is a **measurable partition** of Ω if $\mathcal{F}' \subset \mathcal{F}$.
N. B. An information of an agent can be described by a measurable partition of Ω.

DEFINITION 9.8: Let $(\Omega, \mathcal{F})$ be a measurable space. A mapping $\mu : \mathcal{F} \mapsto \Re_+$ is a **measure** if

(1) $\mu(\emptyset) = 0$,

(2) $A_n \in \mathcal{F}, \forall i \in \mathcal{N}$ with $A_i \cap A_j = \emptyset, \forall i \neq j \Rightarrow \mu(\cup_{n \in N} A_n) = \sum_{n \in N} \mu(A_n)$.

$(\Omega, \mathcal{F}, \mu)$ is called a **measure space**.

NOTE μ is a **probability measure** with additional condition $\mu(\Omega) = 1$, and $(\Omega, \mathcal{F}, \mu)$ is called a **probability space**.