lower sections, then there exists a fixed point of φ.

THEOREM 8.3: Let X be a nonempty compact convex subset of R^ℓ and a correspondence $\varphi : X \to 2^X$ be nonempty-valued and convex-valued. If φ is lower hemi-continuous, then there exists a fixed point of φ.

THEOREM 8.4 (Kakutani): Let X be a nonempty compact convex subset of R^ℓ and the correspondence $\varphi : X \to 2^X$ be nonempty-valued and convex-valued. If φ has closed graph (or is closed-valued and upper hemi-continuous), then there exists a fixed point of φ.

N. B. Note that Theorems 6.1 is a corollary of Theorem 6.4, and that Theorem 6.2 is a corollary of Theorem 6.3.

9 Probability

DEFINITION 9.1: Ω is a **state space** (set of states of nature). $A \subset \Omega$ is an **event**.

DEFINITION 9.2: A family $\mathcal{F}$ of subsets of Ω is a σ-**algebra** if

(1) $\Omega \in \mathcal{F}$,

(2) $A \in \mathcal{F} \Rightarrow A^c \in \mathcal{F}$,

(3) $A_n \in \mathcal{F}, \forall n \in \mathbf{N} \Rightarrow \bigcup_{n \in N} A_n \in \mathcal{F}$.

The pair $(\Omega, \mathcal{F})$ is called a **measurable space**.

DEFINITION 9.3 : Let $\mathcal{A}$ be a family of subsets of Ω. We denote by $\sigma(\mathcal{A})$ the smallest σ-field containing $\mathcal{A}$.

DEFINITION 9.4: For a topological space (X, τ) , $\mathcal{B}(X) := \sigma(\tau)$ is the **Borel σ-field** on X.

Example

(1) 2^Ω is a σ-field.

(2) $\Omega = \{\omega_1, \omega_2, \omega_3\}$. $\mathcal{F} = \{\{\omega_1, \omega_2\}, \{\omega_3\}, \emptyset, \Omega\}$ is a σ-field.