7 Selection Theorems

DEFINITION 7.1: A **selection** from $\varphi : X \to 2^Y$ is a function $f : X \to Y$ such that, for every $x \in X$, $f(x) \in \varphi(x)$. If X and Y are topological spaces, then we say that f is a **continuous selection** if f is a selection and is continuous.

THEOREM 7.2(Yannelis-Prabhakar): Let X be paracompact[3], Y be topological vector space a correspondence $\varphi : X \to 2^Y$ be nonempty-valued and convex-valued. If φ has open lower sections, then there exists a continuous selection of φ.

THEOREM 7.3(Michael): Let X be paracompact, Y be separable Banach space [4] and a correspondence $\varphi : X \to 2^Y$ be nonempty-valued and convex-valued. If φ is lower hemi-continuous, then there exists a continuous selection of φ.

N. B.

(a) If $Y = R^n$ then Theorem 5.1 is a corollary of Theorem 5.2.

(b) If Y is any arbitrary linear topological space then Theorem 5.1 does not follow from Theorem 5.2.

8 Fixed Point Theorems

DEFINITION 8.1:

(1) Let $f : X \to X$ be a function. A **fixed point** of f is a point $x^* \in X$ such that $x^* = f(x^*)$.

(2) Let $\varphi : X \to 2^X$ be a correspendence. A **fixed point** of φ is a point x^* such $x^* \in \varphi(x^*)$.

THEOREM 8.1 (Brouwer): Let X be a nonempty compact convex subset of R^ℓ and $f : X \to X$ be a continuous function. Then there exists a fixed point of f.

THEOREM 8.2 (Browder): Let X be a nonempty compact convex subset of R^ℓ and a correspondence $\varphi : X \to 2^X$ be nonempty-valued and convex-valued. If φ has open

[3]A Hausdorff space is paracompact if every cover has an open locally finite refinement cover

[4]A Banach space is a normed space that is also a complete metric space under the metric induced by its norm