(1) The "value function" $m : X \to \Re$ defined by $m(x) = \sup\{f(x,y) : y \in \varphi(x)\}$ is continuous and

(2) The correspondence $\mu : X \to 2^Y$ defined by

$$\mu(x) = \{y \in \varphi(x) : f(x,y) = m(x)\}$$

is upper hemicontinuous with nonempty, compact-valued.

6 KKM Theorem, Existence of Maximal Element

THEOREM 6.1(Knaster-Kuratowski-Mazurkewicz) : Let X be an arbitrary convex subset of R^l. For $x \in X$, let $F(x)$ be a closed set in R^l satisfying the following assumptions:

(1) For any arbitrary set of points $\{x_1, \ldots, x_n\}$ of X,

$$co\{x_i, \ldots, x_n\} \subset \bigcup_{i=1}^{n} F(x_i).$$

(2) $F(x)$ is compact for at least one $x \in X$.

Then $\bigcap_{x \in X} F(x) \neq \emptyset$.

THEOREM 6.2(Existence of Maximal Element) : Let X be a nonempty compact convex subset of R^ℓ and a correspondence $P : X \to 2^X$ be a preference correspondence such that:

(1) $x \notin P(x)$ for all $x \in X$

(2) $P(x)$ is convex for all $x \in X$

(3) P has open lower sections

Then there exists $x' \in X$ such that $P(x') = \emptyset$.

NOTE (1),(2) can be replaced by $x \notin conP(x)$ for all $x \in X$.

REMARK
KKM Theorem ⇔ Browder Fixed Point Theorem ⇔ Existence of Maximal Elements Theorem