(2) Let $\varphi(x) = (0, 1)$. Then it is upper hemi-continuous but does not have closed graph.

THEOREM 4.7 (Closure): Let $\varphi : X \to 2^Y$ be a correspondence. Define the correspondence $\bar{\varphi} : X \to 2^Y$ by $\bar{\varphi}(x) = \bar{\varphi(x)}$.

(1) If φ is upper hemi-continuous at x, so is $\bar{\varphi}$.

(2) φ is lower hemi-continuous at x iff $\bar{\varphi}$ is lower hemi-continuous at x.

EXAMPLE 4.3: Consider $\varphi : R \to 2^R$ with $\varphi(x) = \{x\}^c$. It is not upper hemi-continuous but its closure is upper hemi-continuous.

THEOREM 4.8 (Intersection): Let φ, μ, and φ_i's be correspondences from X to Y. Define the correspondence $\bigcap_{i \in I} \varphi_i : X \to 2^Y$ by $(\bigcap_{i \in I} \varphi_i)(x) = \bigcap_{i \in I} \varphi_i(x)$. Suppose the intersection is nonempty-valued.

(1) $G_{\cap \varphi_i} = \bigcap_{i \in I} G_{\varphi_i}$.

(2) If φ_i is closed-valued and upper hemi-continuous at x for every $i \in I$, so is $\bigcap_i \varphi_i$.

(3) If φ is lower hemi-continuous at x and μ has open graph, then $\varphi \cap \mu$ is lower hemi-continuous at x.

(4) If φ and μ have open (closed) sections, so does $\varphi \cap \mu$.

THEOREM 4.9 (Union): Let $\varphi_i : X \to 2^Y$ be a correspondence for every $i \in I$. Define the correspondence $\bigcup_{i \in I} \varphi_i : X \to 2^Y$ by $(\bigcup_{i \in I} \varphi_i)(x) = \bigcup_{i \in I} \varphi_i(x)$.

(1) $G_{\cup \varphi_i} = \bigcup_{i \in I} G_{\varphi_i}$.

(2) If φ_i is upper hemi-continuous (closed) at x for every $i = 1, \ldots, n$, so is $\bigcup_i \varphi_i$.

(3) If φ_i is lower hemi-continuous at x for every $i \in I$, so is $\bigcup_i \varphi_i$.

THEOREM 4.10 (Composition): Let $\varphi : X \to 2^Y$ and $\mu : Y \to 2^Z$ be correspondences. Define the correspendence $\mu \circ \varphi : X \to Z$ by $(\mu \circ \varphi)(x) = \bigcup_{y \in \varphi(x)} \mu(y)$.

(1) If φ and μ are upper semi-continuous at x, so is $\mu \circ \varphi$.

(2) If φ and μ are lower semi-continuous at x, so is $\mu \circ \varphi$.