(1) Let φ be compact-valued and upper hemi-continuous. If K is compact, then $\varphi(K)$ is compact (closed).

(2) If φ has open (closed) graph, then it has open (closed) sections.

(3) If φ has open lower sections, then it is lower hemi-continuous.

(4) If φ has open graph, then it is lower hemi-continuous.

(5) If φ is singleton-valued at x and either upper hemi-continuous or lower hemi-continuous at x, then it is continuous at x.

N. B. Note that (4) is a corollary of (2) and (3).

THEOREM 4.5: An upper hemicontinous correspondence $\varphi : X \to 2^Y$ is closed if either:

(1) φ is closed-valued and Y is regular [1], or

(2) φ is compact-valued and Y is Hausdorff [2].

For a correspondence having a compact Hausdorff range, the properties of being closed and being upper hemicontinous coincide.

THEOREM 4.6(Closed Graph Theorem): A closed-valued correspondence with compact Hausdorff range is closed if and only if it it upper hemicontinuous.

Similarly, φ is lower hemi-continuous at x iff $x_n \to x$ and $y \in \varphi(x)$ imply that there exists a sequence $\{y_n\}$ such that $y_n \in \varphi(x_n)$ for every n and $y_n \to y$.

EXAMPLE 4.2: Consider a correspondence $\varphi : R \to 2^R$.

(1) Define φ by

$$\varphi(x) = \begin{cases} 1/x & \text{if } x > 0 \\ \{0\} & \text{if } x = 0 \end{cases}$$

Then φ has closed graph and compact-valued, but is not upper hemi-continuous.

[1] A topological space is **regular** if every nonempty closed set and every singleton disjoint from it can be separated by open sets

[2] A topology is called **Hausdorff** if any two distinct points can be separated by disjoint neighborhood of the points. That is, for each pair $x, y \in X$ with $x \neq y$ there exist neighborhoods $U \in N_x$ and $V \in N_y$ such that $U \cap V = \emptyset$