x is the upper contour set of $\succ$ at x and the lower section $P^{-1}(x)$ of P at x is the lower contour set of $\succ$ at x.

DEFINITION 4.5: Let $\varphi : X \to 2^Y$ be a correspondence.

(1) φ is **closed at** x if $(x_n, y_n) \to (x, y)$ and $y_n \in \varphi(x_n)$ for every n imply $y \in \varphi(x)$. It is **closed** (**has closed graph**) if it is closed at every $x \in X$.

(2) φ is **upper hemi-continuous (u.h.c.) at** x if, for every open set V containing $\varphi(x)$, there exists a neighborhood U of x such that $\varphi(x') \subset V$ for every $x' \in U$. φ is **upper hemi-continuous** if it is upper hemi-continuous at every $x \in X$.

(3) φ is **lower hemi-continuous (l.h.c.) at** x if, for every open set V with $\varphi(x) \cap V \neq \emptyset$, there exists a neighborhood U of x such that $\varphi(x') \cap V \neq \emptyset$ for every $x' \in U$. A correspondence φ is **lower hemi-continuous** if it is lower hemi-continuous at every $x \in X$.

(4) φ is **continuous at** x if φ is both upper hemi-continuous and lower hemi-continuous at x. It is **continuous** if it is continuous at every $x \in X$.

THEOREM 4.2: Let $\varphi : X \to 2^Y$ be a correspondence. The following are equivalent.

(1) φ is upper hemi-continuous.

(2) For each open subset B of Y, $\varphi^+(B)$ is open.

(3) For each closed subset C of Y, $\varphi^-(C)$ is closed.

THEOREM 4.3: Let $\varphi : X \to 2^Y$ be a correspondence. The following are equivalent.

(1) φ is lower hemi-continuous.

(2) For each open subset B of Y, $\varphi^-(B)$ is open.

(3) For each closed subset C of Y, $\varphi^+(C)$ is closed.

COROLLARY : Let $\varphi : X \to 2^Y$ be a correspondence.

(1) If φ is upper hemi-continuous, then $\{x \in X : \varphi(x) \neq \emptyset\}$ is closed.

(2) If φ is lower hemi-continuous, then $\{x \in X : \varphi(x) \neq \emptyset\}$ is open.

THEOREM 4.4: Let $\varphi : X \to 2^Y$ be a correspondence.