DEFINITION 4.2: The **graph** of a correspondence $\varphi : X \to 2^Y$ is defined by

$$G_\varphi = \{(x, y) \in X \times Y : y \in \varphi(x)\}.$$

A correspondence $\varphi : X \to 2^Y$ **has open (closed) graph** if the set $G := \{(x, y) \in X \times Y : y \in \varphi(x)\}$ is open (closed) in $X \times Y$.

DEFINITION 4.3: Let $\varphi : X \to 2^Y$ be a correspondence, $A \subset X$, and $B \subset Y$.

(1) The **image** of A by φ is defined by $\varphi(A) = \bigcup_{x \in A} \varphi(x)$.

(2) The **inverse** of B by φ is defined by $\varphi^{-1}(B) = \{x \in X : \varphi(x) = B\}$.

(3) The **upper inverse** of B by φ is defined by $\varphi^{+}(B) = \{x \in X : \varphi(x) \subset B\}$.

(4) The **lower inverse** of B by φ is defined by $\varphi^{-}(B) = \{x \in X : \varphi(x) \cap B \neq \emptyset\}$.

(5) The **upper section** of φ at x is defined by $\varphi(x)$.

(6) The **lower section** of φ at y is defined by $\varphi^{-1}(y) = \{x \in X : y \in \varphi(x)\}$.

NOTE : $\varphi^{-1}(y) = \varphi^{-}(\{y\})$.

THEOREM 4.1: Let $\varphi : X \to 2^Y$ be a correspondence and $B \subset Y$.

(1) $\varphi^{-1}(B) \subset \varphi^{+}(B) \subset \varphi^{-}(B)$.

(2) $\varphi^{+}(B^c) = [\varphi^{-}(B)]^c$.

(3) $\varphi^{-}(B) = \bigcup_{y \in B} \varphi^{-}(\{y\}) = \bigcup_{y \in B} \varphi^{-1}(y)$.

DEFINITION 4.4: Let $\varphi : X \to 2^Y$ be a correspondence.

(1) φ **has open (closed) upper sections** if $\varphi(x)$ is open (closed) for every $x \in X$.

(2) φ **has open (closed) lower sections** if $\varphi^{-1}(y)$ is open (closed) for every $y \in Y$.

(3) φ **has open (closed) sections** if it has both open (closed) upper sections and open (closed) lower sections.

N. B. φ has open (closed) upper sections iff φ is open-valued (closed-valued).

EXAMPLE 4.1: Define the correspondence $P : X \to 2^X$ by $P(x) := \{x' \in X : x' \succ x\}$ and $P^{-1} : X \to 2^X$ by $P^{-1}(x) := \{x' \in X : x \succ x'\}$. The upper section $P(x)$ of P at