by $intS$.

DEFINITION 2.8: A **neighborhood** U of $x \in R^\ell$ is a subset which contains an open set B containing x. A **neighborhood** U of $S \subset R^\ell$ is a subset which contains an open set B containing S.

DEFINITION 2.9: A subset S of R^ℓ is **closed** if its complement is an open set.

DEFINITION 2.10: A subset S of R^ℓ is **closed relative to (in)** X if there exists a closed subset A of R^ℓ such that $S = A \cap X$.

DEFINITION 2.11: A point $x \in R^\ell$ is a **closure point (adherent point)** of $S \subset R^\ell$ if every open ball at x contains at least one element of S. The set of all closure points of S is the closure of S and is denoted by $\bar{S}$.

DEFINITION 2.12: A point $x \in R^\ell$ is an **accumulation point (cluster point, limit point)** of $S \subset R^\ell$ if every open ball at x contains one element of S which is distinct from x. The set of all accumulation points of S is the **derived set** of S and is denoted by S'.

N. B. Note that x need not be an element of S. Clearly, every accumulation point of a set must be a closure point of that set. It should be clear that $\bar{S} = S \cup S'$. In particular, it follows that a set is closed iff it contains its accumulation points.

DEFINITION 2.13: A sequence $\{x_n\}$ in R^ℓ is **convergent** to x in R^ℓ if

$$\lim_{n\to\infty} d(x_n, x) = 0$$

We write $\lim_n x_n = x$ or $x_n \to x$.

THEOREM 2.3: Let S be a subset of R^ℓ. Then a point $x \in R^\ell$ belongs to $\bar{S}$ iff there exists a sequence $\{x_n\}$ of S such that $x_n \to x$. In particular, if x is an accumulation point of S, then there exists a sequence of S with distinct terms that converges to x.

N. B. A subset S of R^ℓ is closed iff the limit of every convergent sequence in S belongs to S.

DEFINITION 2.14: A point x is a **boundary point** of $S \subset R^\ell$ if every open ball of x has a nonempty intersection with S and $R^\ell \setminus S$. The set of all boundary points of S