(6) $R^\ell_- = \{x \in R^\ell : x \leq 0\}$.

(7) The **sum** of sets $X_1 \subset R^\ell$, $X_2 \subset R^\ell$ is defined by $X_1 + X_2 = \{x_1 + x_2 : x_i \in X_i, i = 1, 2\}$.

(8) Let $\alpha \in R$ and $X \subset R^\ell$. $\alpha X = \{\alpha x : x \in X\}$.

(9) The **product** of sets $X_1 \subset R^\ell$, $X_2 \subset R^\ell$ is defined by $\prod_{i=1}^2 X_i = X_1 \times X_2 = \{(x_1, x_2) : x_i \in X_i, i = 1, 2\}$.

(10) The **dot product** of x and y is definded by $x \cdot y = \sum_{k=1}^\ell x_k y_k$.

(11) The **Euclidean norm** $\|x\|$ of x is defined by $\|x\|^2 = x \cdot x$.

THEOREM 2.2: Let X and Y be sets and let $f : X \to Y$ be a function. Let A and A_i's be subsets of X, and B and B_i's be subsets of Y. Then the following hold:

(1) $\bigcap_{i \in I} f^{-1}(B_i) = f^{-1}(\bigcap_{i \in I} B_i)$.

(2) $\bigcup_{i \in I} f^{-1}(B_i) = f^{-1}(\bigcup_{i \in I} B_i)$.

(3) $X \setminus f^{-1}(B) = f^{-1}(Y \setminus B)$.

(4) $\bigcup_{i \in I} f(A_i) = f(\bigcup_{i \in I} A_i)$.

(5) $f(\bigcap_{i \in I} A_i) \subset \bigcap_{i \in I} f(A_i)$.

(6) $f(f^{-1}(B)) \subset B$, and $f(f^{-1}(B)) = B$ iff f is onto.

(7) $A \subset f^{-1}(f(A))$, and $A = f^{-1}(f(A))$ iff f is one-to-one.

DEFINITION 2.4: If $x \in R^\ell$, then the **open ball** at x with radius $\varepsilon > 0$ is the set

$$B_\varepsilon(x) = \{x' \in R^\ell : d(x, x') < \varepsilon\}.$$

DEFINITION 2.5: A subset S of R^ℓ is **open** if for every $x \in S$, there exists a open ball $B_\varepsilon(x) \subset S$.

DEFINITION 2.6: A subset S of R^ℓ is **open relative to (in)** X if there exists an open subset A of R^ℓ such that $S = A \cap X$.

DEFINITION 2.7: A point $x \in R^\ell$ is an **interior point** of $S \subset R^\ell$ if there exists an open ball $B_\varepsilon(x) \subset S$. The set of all interior points of S is the **interior** of S and is denoted