Part I

MATHEMATICS

1 Topological Space

DEFINITION 1.1: A **topology** τ on a set X is a collection of subsets of X satisfying:

(1) $\emptyset, X \in \tau$,

(2) τ is closed under finite intersections.

(3) τ is closed under arbitrary unions.

The pair (X, τ) is called a **topological space**. We call a member of τ an **open set** in X. The complement of an open set is a **closed set**.

DEFINITION 1.2: Let (X, τ) be a topological space, and let A be any subset of X.

(1) The **interior** of A, denoted by $intA$, is the largest open set included in A.

(2) The **closure** of A, denoted by $\bar{A}$, is the smallest closed set including A.

(3) A **neighborhood** of a point x is any set V containing x in its interior. In this case we say that x is an **interior point** of V.

(4) A point x is **closure point** of the set A if every neighborhood of x meets A. Note that $\bar{A}$ coincides with the set of all closure points of A.

(5) A point x is **accumulation point** (or a **limit point**, or a **cluster point**) of the set A if every neighborhood V of x we have $(V \setminus \{x\}) \cap A \neq \emptyset$. The set of all accumulation points of A is denoted by A'.

(6) A point x is a **boundary point** of A if each neighborhood V of x satisfies both $V \cap A \neq \emptyset$ and $V \cap A^c \neq \emptyset$. The set of all boundary points of A is denoted by ∂A.

DEFINITION 1.3: A function $f : X \to Y$ between two topological spaces is **continuous** if $f^{-1}(U)$ is open for every open set U. We say that f is **continuous at the point** x if $f^{-1}(V)$ is a neighborhood of x whenever V is a neighborhood of $f(x)$.