

Ideal Bipolar Correlation Property of Pilot Bit Patterns in W-CDMA System

Young Joon Song

Department of Electronic Engineering
Kumoh National Institute of Technology,
1 Yangho-dong, Gumi, Gyungbuk, 730-701, Korea
yjsong@kumoh.ac.kr

Abstract. This paper shows that the pilot bit patterns in W-CDMA (Wideband Code Division Multiple Access) system have ideal bipolar correlation property. The sequences have the lowest sidelobes except at the middle shift and have the property of odd-periodic sequence. The sequences can be inserted into a bit stream to align the timing at the transmitter and the receiver. Using the proposed method, we can generate the sequences of length 30 with ideal bipolar correlation property using the ideal auto-correlation of the pilot bit patterns of length 15 in W-CDMA system.

Keywords: Synchronization, Bipolar correlation, Pilot bit pattern, W-CDMA system

1 Introduction

In 3GPP (3rd Generation Partnership Project) W-CDMA (Wideband Code Division Multiple Access) system of IMT-2000 technology, the FSW (Frame Synchronization Word) of time multiplexed pilot bits are used for frame synchronization confirmation [1]-[3]. In this paper, we show that the pilot bit patterns in W-CDMA system have ideal bipolar correlation property. We can construct odd-periodic sequences of length 30 with optimal synchronization property using the ideal auto-correlation of the pilot bit patterns of length 15 in W-CDMA system. The odd-periodic sequence has double maximum correlation values that are equal in magnitude and opposite in polarity at the zero and middle shifts. As the pilot bit patterns are based on binary maximal length sequences, we can construct odd-periodic binary sequences with lowest sidelobes using the ideal auto-correlation of the patterns. This special ideal bipolar correlation property can be useful in designing the synchronization circuit at a receiver.

2 Definitions

The periodic auto-correlation function is defined as

$$R_i(\tau) = \sum_{t=0}^{n-1} (-1)^{s_{i,t} + s_{i,[(t+\tau) \bmod n]}} \quad (1)$$

where $\bmod n$ denotes modulo n and $s_{i,t} + s_{i,[(t+\tau) \bmod n]}$ is computed modulo 2. We call $R(0)$ the “in-phase auto-correlation value” and all other $R(\tau)$ ($\tau \neq 0$) the “out-of-phase auto-correlation values.” Sequence \mathbf{S} is said to possess the “ideal auto-correlation property” if its periodic auto-correlation function has the lowest out-of-phase auto-correlation value of “1” or “-1,” which is given by

$$R_s(\tau) = \begin{cases} n, & \tau \equiv 0(\bmod n) \\ -1, & \tau \neq 0(\bmod n) \end{cases} \quad (2)$$

$$R_s(\tau) = \begin{cases} n, & \tau \equiv 0(\bmod n) \\ 1, & \tau \neq 0(\bmod n) \end{cases} \quad (3)$$

Let $\mathbf{S} = (s_i)$ be a binary sequence of period n . Periodic repetition, but with the reversal of the signs of alternate periods, gives the odd-periodic sequence $\mathbf{A} = (a_i)$ [4]. This means that

$$a_i = \begin{cases} s_i, & 0 \leq i \leq n-1 \\ \bar{s}_i, & n \leq i \leq 2n-1 \end{cases} \quad (4)$$

where $\bar{s}_i = (s_i + 1) \bmod 2$. The odd-periodic sequence has double maximum correlation values that are equal in magnitude and opposite in polarity at the zero and middle shifts. In this paper, the sidelobe of an odd-periodic sequence is defined as the out-of-phase auto-correlation value except at the middle shift.

3 Pilot Bit Patterns

$\mathbf{S}_i = (s_{i,0}, s_{i,1}, \dots, s_{i,14})$, $i = 1, \dots, 4$, of Table 1 are pilot bit patterns in W-CDMA defined as frame synchronization word [1]-[2]. Two patterns \mathbf{S}_1 and \mathbf{S}_3 are binary maximal length sequences generated from the polynomials

$$X(x) = x^4 + x^3 + 1 \quad (5)$$

$$Y(x) = x^4 X(x^{-1}) = x^4 + x + 1 \quad (6)$$

with initial values of shift registers as (0,0,0,1) and (0,0,1,1), respectively [2]. And S_2 and S_4 are the complement of the 8 times cyclic shifted version of S_1 and S_3 , respectively.

Table 1. Pilot Bit Patterns in W-CDMA system

Pilot bit patterns	
$S_1 = (1\ 0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 0)$	$S_2 = (1\ 0\ 1\ 0\ 0\ 1\ 1\ 1\ 0\ 1\ 1\ 1\ 0\ 0\ 0\ 0)$
$S_3 = (1\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 1\ 0\ 1\ 0\ 1\ 1)$	$S_4 = (0\ 0\ 1\ 0\ 1\ 0\ 0\ 0\ 0\ 1\ 1\ 1\ 0\ 1\ 1)$

4 Optimal Synchronization Sequences

In this section we present a method to construct optimal odd-periodic synchronization sequences with lowest sidelobes. On the basis of the ideal auto-correlation property of sequences with an odd length n , a new method of constructing optimal odd-periodic synchronization sequences of length $2n$ is proposed. The auto-correlation of a sequence has double maximum values equal in magnitude and opposite in polarity at zero and middle shifts with the lowest out-of-phase values excluding those at the middle shift.

For example, let $\mathbf{S} = (s_0, s_1, s_2, s_3, s_4) = (0, 0, 0, 0, 1)$ with ideal auto-correlation property:

$$R_s(\tau) = \begin{cases} 5, & \tau \equiv 0 \pmod{5} \\ 1, & \tau \not\equiv 0 \pmod{5} \end{cases} \quad (7)$$

And consider a sequence $\mathbf{A} = (a_i)$ of length $N = 2n = 10$ with the bit mapping:

$$\begin{aligned} a_0 &= s_0 = 0 & a_1 &= \bar{s}_3 = 1 \\ a_2 &= s_1 = 0 & a_3 &= \bar{s}_4 = 0 \\ a_4 &= s_2 = 0 & a_5 &= \bar{s}_0 = 1 \\ a_6 &= s_3 = 0 & a_7 &= \bar{s}_1 = 1 \\ a_8 &= s_4 = 1 & a_9 &= \bar{s}_2 = 1 \end{aligned}$$

Then $\mathbf{A} = (0, 1, 0, 0, 0, 1, 0, 1, 1, 1)$ becomes the odd-periodic sequence with lowest sidelobes and the ideal bipolar correlation property becomes

$$R_a(\tau) = \begin{cases} 10, \tau \equiv 0(\text{mod}10) \\ -10, \tau \equiv 5(\text{mod}10) \\ -2, \tau = \text{odd}, \tau \neq 5(\text{mod}10) \\ 2, \tau = \text{even}, \tau \neq 0(\text{mod}10) \end{cases} \quad (8)$$

Two pairs of pilot bit patterns [S_1, S_2] and [S_3, S_4] have the relationships (9) and (10), respectively.

$$\bar{s}_{1,(z+8)(\text{mod}n)} = s_{2,z} \quad (9)$$

$$\bar{s}_{3,(z+8)(\text{mod}n)} = s_{4,z} \quad (10)$$

As illustrated in the example, from the pilot bit patterns we can construct odd-periodic sequences **A** and **B** of length $N = 2n = 30$ with the mapping:

$$a_{2z} = s_{1,z}, \quad a_{2z+1} = s_{2,z}, \quad b_{2z} = s_{3,z}, \quad b_{2z+1} = s_{4,z}$$

Then the auto-correlation function of two sequences **A** and **B** becomes

$$R(\tau) = R_a(\tau) = R_b(\tau) = \begin{cases} 30, \tau \equiv 0(\text{mod}30) \\ -30, \tau \equiv n(\text{mod}30) \\ 2, \tau = \text{odd}, \tau \neq 15(\text{mod}30) \\ -2, \tau = \text{even}, \tau \neq 0(\text{mod}30) \end{cases} .$$

This special ideal bipolar correlation property of the pilot bit patterns can give more flexibility in the design of a synchronization circuit.

References

1. 3GPP Technical Specification 25.211: Physical Channels and Mapping of Transport Channels onto Physical Channels (FDD), Sophia Antipolis Valbonne, France, September (2002)
2. Y. J. Song: Frame Synchronization Word in W-CDMA System, IEEE Comm. Letters, vol. 7, no. 11, pp. 520-522, November (2003)
3. R. A. Scholtz: Frame Synchronization Techniques, IEEE Trans. Comm., vol. 28, no. 4, pp.1204-1213. August (1980)
4. H. D. Luke: Binary Odd-periodic Complementary Sequences, IEEE Trans. Inform. Theory, vol. 43, no. 1, pp. 365-367, January (1997)