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AN OBSTACLE AVOIDANCE METHOD FOR CHAOTIC ROBOTS USING ANGULAR DEGREE LIMITATIONS

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Abstract

This paper presents a method to avoid obstacles that have unstable limit cycles in a chaos trajectory surface using angular degree limits. It is assumed that all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. When a chaos robot meets an obstacle in a Lorenz, Hamilton and Hyper-chaos equation trajectory that exceed the defined angular degree limits, the obstacle repulses the robot.

Computer simulation of the Lorenz equation and the Hamilton and hyper-chaos equation trajectories, with one or more Van der Pol equations as the obstacle(s) is performed and the proposed method is verified through simulation of the chaotic trajectories in any plane, which avoids the obstacle when it is found, where the target is either met or within close range.

Keywords— chaos, mobile robot, Lorenz equations, Hamilton equations, Hyper-chaos equations, obstacle avoidance

1. Introduction

Chaos theory has drawn a great deal of attention in the scientific community for almost two decades. Considerable research efforts have been performed over the past few years to export these physics and mathematics concepts into real world engineering applications. Applications of chaos are being actively investigated in such areas as chaos control [1]-[2], chaos synchronization and secure/crypto communication [3]-[7], chemistry [8], biology [9] and related robotic applications [10].

Recently, Nakamura, Y. et al [10] proposed a chaotic mobile robot which was equipped with a controller to ensure chaotic motion, and whose dynamics are represented by an Arnold equation. This investigation applied obstacles in the chaotic trajectory, but did not address chaos obstacle avoidance methods.

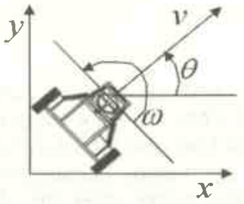
This paper proposes a method for target searching using unstable limit cycles in the chaos trajectory surface. It is assumed that all obstacles in the chaos trajectory surface have a Van der Pol equation with an unstable limit cycle. When the method identifies the target, through arbitrary wandering in the chaos trajectories derived from Lorenz, Hamilton and hyper-chaos trajectory equations, it is applied to the chaos robots.

The computer simulations show multiple obstacle avoidance with Lorenz and Hamilton equations and hyper-chaos equations. The proposed method avoids the obstacle, within a defined range, and the results are verified to demonstrate application of the target search, with chaotic trajectories in any plane, to the mobile robot.

2. Chaotic Mobile Robot Equations

Mobile robot

A two- wheeled mobile robot is defined as the following mobile robot mathematical model shown in (Fig. 1).



(Fig. 1) two-wheeled mobile robot

Let the linear velocity of the robot v [m/s] and angular velocity ω[rad/s] be the inputs in the system. The state equation of the two-wheeled mobile robot is written as follows:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{pmatrix} = \begin{pmatrix} \cos\theta & 0 \\ \sin\theta & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} v \\ \omega \end{pmatrix} \quad (1)$$

where (x,y) is the position of the robot and θ is the angle of the robot.

Chaos equations
In order to generate chaotic motions for the mobile robot, chaos equations such as Lorenz, Hamilton and hyper-chaos equations, are applied.

Lorenz equation

The Lorenz equation is defined as:

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = \gamma x - y - xz \quad (2)$$
$$\dot{z} = xy - bz$$

where $\sigma = 10, r = 28, b = 8/3$. The Lorenz equation describes the famous chaotic phenomenon.

Hamilton equation

The Hamilton equation is one of the simplest physical models and has been widely investigated through mathematical, numerical and experimental methods. The state equation of the Hamilton equation is derived as follows.

$$\dot{x}_1 = x_1(13 - x_1^2 - y_1^2)$$
$$\dot{x}_2 = 12 - x_1(13 - x_1^2 - y_1^2) \quad (3)$$

Hyper-chaos equation

Hyper-chaos equations are also one of the simplest physical models and have been widely investigated by mathematical, numerical and experimental methods for complex chaotic dynamics. The hyper-chaotic equation is developed by using a connected N-double scroll. The state equation of N-double scroll equation is derived as follows:

$$\dot{x} = a[y - h(x)]$$
$$\dot{y} = x - y + z \quad (4)$$
$$\dot{z} = -\beta y$$

The hyper-chaos equation is composed from a 1 dimensional CNN (Cellular Neural Network), with two identical N-double scroll circuits. Each cell is connected by using unidirectional or diffusive coupling. This paper uses the diffusive coupling method and the state equation of x-diffusive coupling and y-diffusive coupling is represented as follows.

x-diffusive coupling

$$\dot{x}^{(j)} = a[y^{(j)} - h(x)^{(j)}] + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)})$$
$$\dot{y}^{(j)} = x^{(j)} - y^{(j)} + z^{(j)} \quad (5)$$
$$\dot{z}^{(j)} = -\beta y^{(j)}, j = 1,2..L$$

y-diffusive coupling

$$\dot{x}^{(j)} = a[y^{(j)} - h(x)^{(j)}]$$
$$\dot{y}^{(j)} = x^{(j)} - y^{(j)} + z^{(j)} + D_y(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \quad (6)$$
$$\dot{z}^{(j)} = -\beta y^{(j)}, j = 1,2....L$$

where, L is number of cell.

Embedding chaos trajectories
In order to embed the chaos equation into the mobile robot, the Lorenz, Hamilton and hyper-chaos equation are utilized as described in the following section.

Lorenz equation

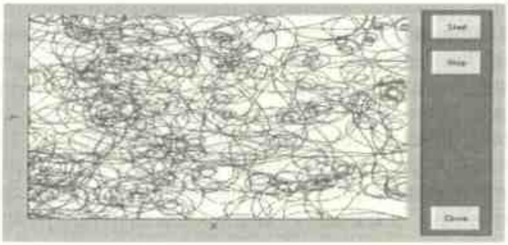
By combining equations (1) and (2), the following state variables are defined:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} \sigma(y - x) \\ \gamma x - y - xz \\ xy - bz \\ v\cos x_3 \\ v\sin x_3 \end{pmatrix} \quad (7)$$

Eq. (7) includes the Lorenz equation. The behavior of the Lorenz equation is chaotic. The chaotic mobile robot trajectory shown in (Fig. 2) may be obtained by using Eq. (7) with coefficient and initial conditions as follows:

Coefficients: v= 1[m/s]
Initial conditions:

$$x_1 = 0.10, \quad x_2 = 0.265, \quad x_3 = 0.27, \quad y = 0.5$$



(Fig. 2) Trajectory of mobile robot of Lorenz equation

Hamilton equation

Through combination of equation (1) and (3), the following state variables are defined (8):

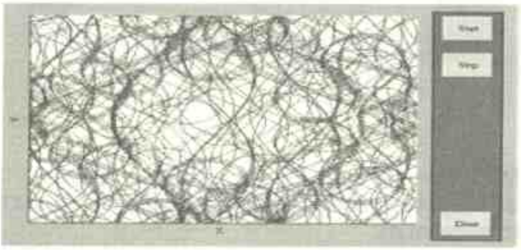
$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} x_1(13 - x_1^2 - y_1^2) \\ 12 - x_1(13 - x_1^2 - y_1^2) \\ v\cos x_3 \\ v\sin x_3 \end{pmatrix} \quad (8)$$

Using equation (8), the chaos robot trajectories using a Hamilton equation are derived. (Fig. 3) shows the phase plane of the Hamilton equation with coefficient and initial conditions as follows:

Coefficients: v= 1[m/s]

Initial conditions:

$x_1 = random, \quad x_2 = random, \quad x = 0.1, \quad y = 0.1$



(Fig. 3) Trajectory of mobile robot of Hamilton equation

Hyper-chaos equation

After combining (1) and (5) or (6), the following state variables are defined (9) or (10):

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} d[y^{(j)} - h(x^{(j)})] + D_x(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ x^{(j)} - y^{(j)} + z^{(j)} \\ -\beta y^{(j)} \\ v\cos x_3 \\ v\sin x_3 \end{pmatrix} \quad (9)$$

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x} \\ \dot{y} \end{pmatrix} = \begin{pmatrix} d[y^{(j)} - h(x^{(j)})] \\ x^{(j)} - y^{(j)} + z^{(j)} + D_y(x^{(j-1)} - 2x^{(j)} + x^{(j+1)}) \\ -\beta y^{(j)} \\ v\cos x_3 \\ v\sin x_3 \end{pmatrix} \quad (10)$$

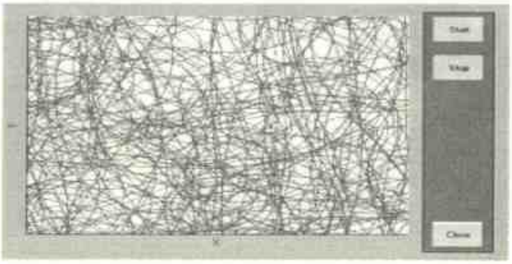
Using equation (8) and (9), the chaos robot trajectories with hyper-chaos equations are generated. (Fig.4) shows the phase plane of the hyper-chaos equation, with coefficient and initial conditions, and is defined as:

Coefficients:

$\alpha = 9, \ \beta = 12.787$, v= 1[m/s], $D_y = 0.01$

Initial conditions:

$x_1 = 0.1, \ x_2 = -0.1 \ x_3 = 0.1, \ x = 0, \ y = 0$



(Fig. 4) Trajectory of mobile robot of Hyper-chaos equation

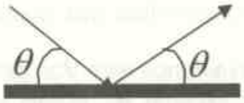
Mirror Mapping

Equations (7) - (10) assume that the mobile robot moves in a smooth state space without boundaries. However, real robots move in space with boundaries like walls or surfaces of targets. To avoid a boundary or obstacle, mirror mapping is applied using (11) and (12). Whenever the robots approach a wall or obstacle, the robot's new position is calculated by using Eq. (11) or (12).

$$A = \begin{pmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{pmatrix} \quad (11)$$

$$A = 1/1+m \begin{pmatrix} 1-m^2 & 2m \\ 2m & -1+m^2 \end{pmatrix} \quad (12)$$

Equation (11) is used when the slope is infinity, such as $\theta = 90$, and equation (12) is used when the slope is not infinity.

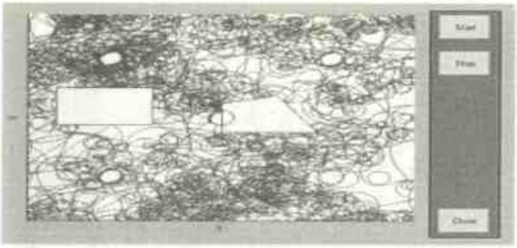


(Fig. 5)Mirror mapping

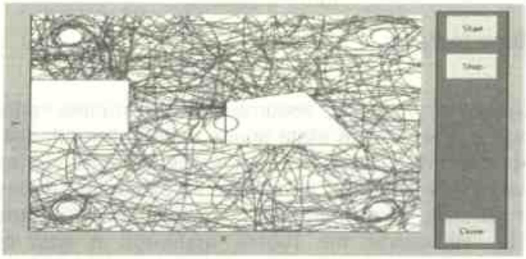
Chaos Robot Behavior with Mirror Mapping and Obstacle Avoidance

In this section, the avoidance behavior of a chaos trajectory with obstacle mapping, relying on the Lorenz, Hamilton and hyper-chaos equation respectively, are presented.

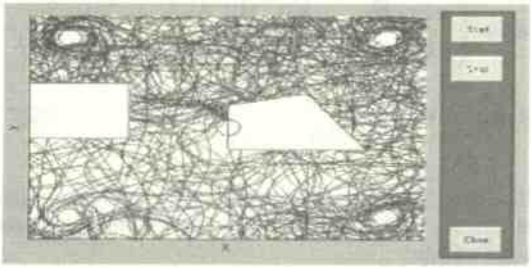
(Fig. 6) through 8 shows the chaos robot trajectories to which mirror mapping is applied in the outer wall and in the inner obstacles and using Eq. (11) and (12), relying on Lorenz (7), Hamilton (8) and hyper-chaos equations (9) or (10). The chaos robot has two fixed obstacles, and it can be confirmed that the robot adequately avoids the fixed obstacles in the Lorenz, Hamilton and Hyper-chaos robot trajectories.



(Fig. 6) Lorenz equation trajectories of chaos robot with obstacle



(Fig.7) Hamilton equation trajectories of chaos robot with obstacle



(Fig.8) Hyper-chaos equation trajectories of chaos robot with obstacles

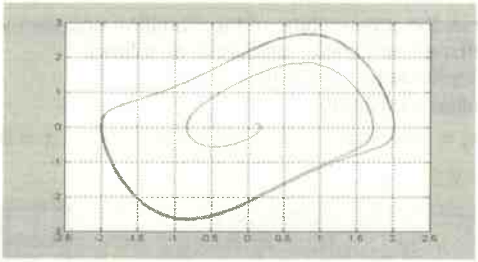
Obstacle Avoidance with Van der Pol Equations

This section presents the mobile robot's avoidance using Van der Pol (VDP) equation obstacles. It is assumed that the target has a VDP equation with a stable limit cycle, because, in this condition, the mobile robot cannot move outside of the VDP target and the target is the searched obstacle.

VDP equation as a hidden obstacle
Obstacles are represented by VDP equations and written as follows:

$$\dot{x} = y$$
$$\dot{y} = (1 - y^2)y - x \qquad (13)$$

From equation (13), the following limit cycle is obtained and shown in (Fig. 9).



(Fig. 9) Limit cycle of VDP

Magnitude of distracting force from the obstacle
The magnitude of distracting force from the Target is represented as:

$$D = \frac{0.325}{(0.2D_k + 1)e^{3(0.2D_k - 1)}} \qquad (14)$$

where D_k is the distance between each effective obstacle and the mobile robot.
The VDP target direction vector is calculated as follows:

$$\begin{bmatrix} \bar{x}_k \\ \bar{y}_k \end{bmatrix} = \begin{bmatrix} x_o - y \\ 0.5(1-(y_o - y)^2)(y_o - y) - (x_o - x) \end{bmatrix} \qquad (15)$$

where (x_o, y_o) are the coordinates of the center point of each target. Then, the magnitude of the VDP direction vector (L), the magnitude of the moving vector of the virtual robot (I) and the enlarged coordinates (I/2L) of the magnitude of the virtual robot in VDP(x_k', y_k') are calculated, as follows:

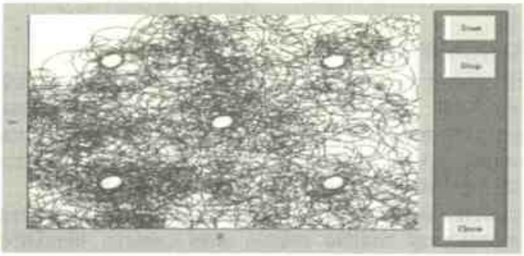
$$L = \sqrt{(\bar{x}_{vdp}^{\,2} + \bar{y}_{vdp}^{\,2})}$$
$$I = \sqrt{(x_r^{\,2} + y_r^{\,2})} \qquad (16)$$
$$x_k' = \frac{\bar{x}_k}{L}\frac{I}{2},\ y_k' = \frac{y_k}{L}\frac{I}{2}$$

Finally, we can get the Total Distraction Vector (TDV) as shown by the following equation:

$$\begin{bmatrix} \dfrac{\sum_k^n ((1 - \frac{D_k}{D_0})\bar{x} + \frac{D_k}{D_0}\bar{x}_k)}{n} \\ \dfrac{\sum_k^n ((1 - \frac{D_k}{D_0})\bar{y} + \frac{D_k}{D_0}\bar{y}_k)}{n} \end{bmatrix} \qquad (17)$$

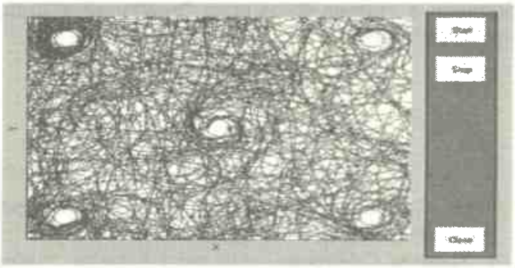
Using equations (14)-(17), the avoidance method of the obstacle in the Lorenz equation, Hamilton and Hyper-chaos equation trajectories with one or more VDP obstacles are calculated..
In Fig. 10, the computer simulation shows that the chaos robot has two robots and a total of 5 VDP obstacles, including two VDP obstacles, at the origin in the Lorenz equation trajectories. The robot

sufficiently avoids the obstacles in the Lorenz equation trajectories.



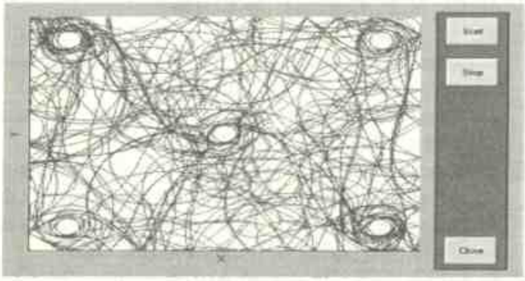
(Fig. 10) Computer simulation result of obstacle avoidance with 3 robots and 5 obstacles in Lorenz equation trajectories.

In (Fig. 11), the computer simulation shows that the chaos robot surface has two robots and total of 5 VDP obstacles, including 2 VDP obstacles at the origin in the Hamilton equation trajectory avoids the obstacles in the Hamilton equation trajectory.



(Fig. 11) Computer simulation result of obstacle avoidance with 3 robots and 5 obstacles in Hamilton equation trajectory.

In (Fig. 12), the computer simulation result shows that the chaos robot surface has two robots and total of 5 VDP obstacles, including 2 VDP obstacles at the origin in the Hyper-chaos equation trajectory, and avoided the obstacles in the Hyper-chaos equation trajectory.



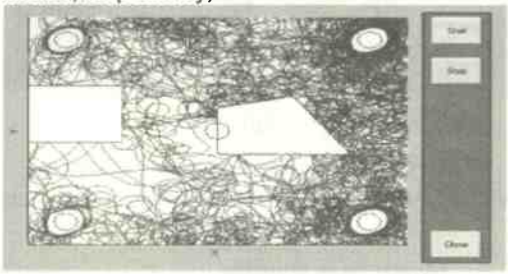
(Fig. 12) Computer simulation result of obstacle avoidance with 3 robots and 5 obstacles in Hyper-chaos equation trajectory

An Obstacle Avoidance Method Using Angular Degree Limits

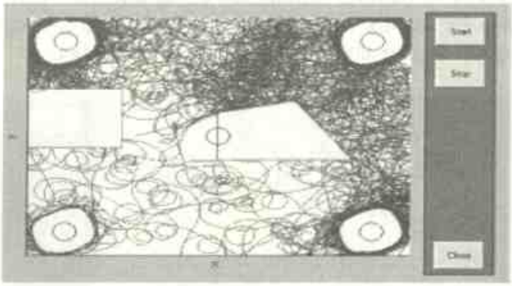
This section proposes a new obstacle avoidance method by using angular degree limits with Lorenz, Hamilton, hyper-chaos equations. This method ensures a safe robot path with distance limits to avoid obstacles. This is done by constraining the limit of the angular degree when approaching obstacles.

Lorenz equation

In (Fig. 13), the robot trajectories for obstacle avoidance through angular degree limitation are demonstrated for (a) low situations and (b), high situation, respectively, in the Lorenz chaos robot.



(a)

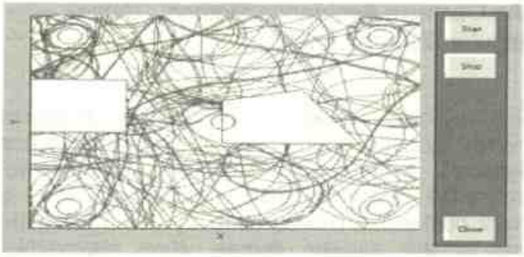


(b)

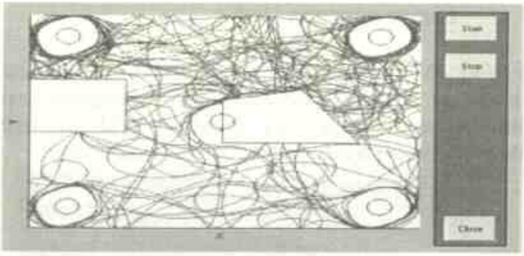
(Fig. 13) Obstacle avoidance results for angular degree limits in the Lorenz chaos robot, low (a), high (b)

Hamilton equation

(Fig. 14) shows the robot trajectories for obstacle avoidance using angular degree limits for (a) low angular degree limits and (b) high angular degree limits in the Hamilton chaos robot, respectively.



(a)

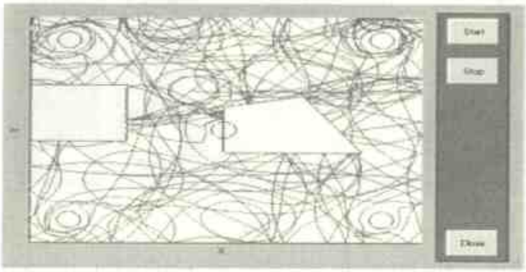


(b)

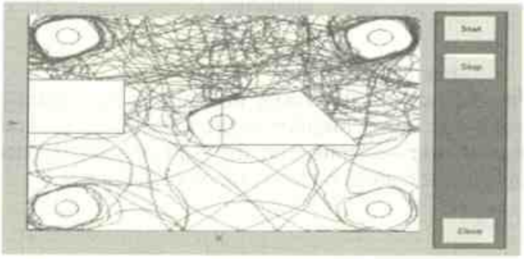
(Fig. 14) An obstacle avoidance result for angular degree limits in the Hamilton equation, low (a), high (b)

Hyper-chaos equation

(Fig. 15), demonstrates the robot trajectories of obstacle avoidance simulation using angular degree limits for (a) low angular degree limits and (b) high angular degree limits in the Hyper-chaos robot.



(a)



(b)

(Fig. 15) Obstacle avoidance for angular degree limits in the Hyper-chaos equation, low limits (a) and high limits(b)

(Figs. 13), (Figs. 14), (Figs. 15), demonstrate the low and high angular degree limits for the obstacle avoidance using angular degree limitations.

Conclusion

This paper proposed a chaotic mobile robot, which employed Lorenz, Hamilton and hyper-chaos equation trajectories, and an obstacle avoidance method in which we assume that the obstacle has a Van der Pol equation with an unstable limit cycle.

Robot trajectories were generated representing the dynamics of mobile robots with Lorenz, Hamilton and hyper-chaos equations and integrating the proposed obstacle avoidance method using angular degree limitations. Computer simulations demonstrated that this method produces successful trajectories through the analysis of angular degree limitations.

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References

[1] E. Ott, C.Grebogi, and J.A York," Controlling Chaos", Phys. Rev.Lett., vol. 64, no.1196-1199, 1990.

[2] T. Shinbrot, C.Grebogi, E.Ott, and J.A.Yorke, " Using small perturbations to control chaos", Nature, vol. 363, pp. 411-417, 1993.

[3] M. Itoh, H. Murakami and L. O. Chua, "Communication System Via Chaotic Modulations" IEICE. Trans. Fundmentals. vol.E77-A, no. , pp.1000-1005, 1994.

[4] L. O. Chua, M. Itoh, L. Kocarev, and K. Eckert, "Chaos Synchronization in Chua's Circuit" J. Circuit. Systems and computers, vol. 3, no. 1, pp. 93-108, 1993.

[5] M. Itoh, K. Komeyama, A. Ikeda and L. O. Chua, " Chaos Synchronization in Coupled Chua Circuits", IEICE. NLP. 92-51. pp. 33-40. 1992.

[6] K. M. Short, " Unmasking a modulated chaotic communications scheme", Int. J. Bifurcation and Chaos, vol. 6, no. 2, pp. 367-375, 1996.

[7] L. Kocarev, " Chaos-based cryptography: A brief overview," IEEE, Vol. pp. 7-21. 2001.

[8] M. Bertram and A. S. Mikhailov, "Pattern formation on the edge of chaos: Mathematical modeling of CO oxidation on a Pt(110) surface under global delayed feedback", Phys. Rev. E 67, pp. 036208, 2003.

[9] K. Krantz, f. H. Yousse, R.W , Newcomb, "Medical usage of an expert system for recognizing chaos", Engineering in Medicine and Biology Society, 1988.

Proceedings of the Annual International Conference of the IEEE, 4-7, pp. 1303 -1304,1988 .
[10] Nakamura, A., Sekiguchi. " The chaotic mobile robot", , IEEE Transactions on Robotics and Automation , Volume: 17 Issue: 6 , pp. 898 -904, 2001.
[11] H. Okamoto and H. Fujii, Nonlinear Dynamics, Iwanami Lectures of Applied Mathematics, Iwanami, Tokyo, 1995, vol. 14.
[12] Y. Bae, J. Kim, Y.Kim, " The obstacle collision avoidance methods in the chaotic mobile robot", 2003 ISIS, pp.591-594, 2003.
[13] Y. Bae, J. Kim, Y.Kim, " Chaotic behavior analysis in the mobile robot: the case of Chua's equation" , Proceeding of KFIS Fall Conference 2003, vol. 13, no. 2, pp.5-8, 2003.
[14] Y. Bae, J. Kim, Y.Kim, " Chaotic behavior analysis in the mobile robot: the case of Arnold equation" , Proceeding of KFIS Fall Conference 2003, vol. 13, no. 2, pp110-113, 2003.
[15] Y. C. Bae, J.W. Kim, Y.I, Kim, Chaotic Behaviour Analysis in the Mobile of Embedding some Chaotic Equation with Obstacle", J.ournal of Fuzzy Logic and Intelligent Systems, vol. 13, no.6호, pp.729-736, 2003.
[16] Y. C. Bae, J. W. Kim, Y.I, Kim, Obstacle Avoidance Methods in the Chaotic Mobile Robot with Integrated some Chaotic Equation", International Journal of Fuzzy Logic and Intelligent System, vol. 3, no. 2. pp. 206-214, 2003.

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