Periodic Polyphase Complementary Sequence Sets Based on Perfect Sequences

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Abstract. A new construction for periodic polyphase complementary sequence sets (PPCSSs) is presented. By making use of sampling a polyphase perfect sequence (PPS) with equal space, the proposed method can produce a PPCSS, and a different PPCSS can be obtained when another PPS is employed, which implies that the family size of the proposed PPCSSs is equivalent to that of the employed PPSs. It is worth mentioning that the number and length of sub-sequences in the proposed PPCSSs can be changed on demand, which illuminates that this new construction is very fit for various engineering applications.

Keywords: complementary sequence, periodic sequence, polyphase sequence, perfect sequence.

1 Introduction

The complementary sequence sets are widely applied to elimination of multiple access interference (MAI) [1], channel estimation [2], synchronization [3], reduction of peak-to-average power ratio (PAPR) [4], and so on. A complementary sequence set possesses an impulsive autocorrelation function, that is, the autocorrelation functions of all its sub-sequences sum to zero except the time shift in the center. Up to now, many construction methods for complementary sequence sets have been presented [5], in which Popović [6] proposed a construction method for aperiodic complementary sequence sets (ACSSs) by making use of polyphase perfect sequences (PPSs) and their cyclic time shifted versions, and the number and length of sub-sequences of the resulting ACSSs are equal to the ones of PPSs employed. In addition, Ref. [5] showed a construction for periodic complementary sequence sets (PCSSs) by using perfect arrays, and the obtained PCSSs possess that the number and length of their sub-

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sequences are the same as the numbers of rows and columns of perfect arrays employed, respectively. It is apparently disadvantageous that in two methods referred to above, the number and length of sub-sequences don't be changed freely, which will result in application obstacle, such as in variable date-rate transmission. In this paper, a new construction method for periodic polyphase complementary sequence sets (PPCSSs) is presented by making use of sampling PPSs with equal space, and in the resulting PPCSSs , the number and length of sub-sequences can be altered according to requirements. As a consequence, such a difficulty mentioned above is conquered to some extent.

2 Basic Concepts

Let $\underline{s}^{(i)} = (s_0^{(i)}, s_1^{(i)}, s_2^{(i)}, \cdots, s_{M-1}^{(i)})$ denote a polyphase sequence with length M , whose elements take the unit magnitude, that is, $|s_k^{(i)}|^2 = 1 \ (0 \le k \le M-1)$. We define the periodic autocorrelation function of the sequence $\underline{s}^{(i)}$ as following.

$$R_{s^{(i)},s^{(i)}}(u) = \sum_{k=0}^{M-1} s_k^{(i)} (s_{k+u}^{(i)})^* \quad (|u| \le M-1), \tag{1}$$

where $(x)^*$ denotes the complex conjugate of x , and the subscript "$k+u$" in (1) is operated modulo M .

Let sequence $\underline{c}$ consist of N sub-sequences, more clearly, $\underline{c} = (\underline{s}^{(0)}, \underline{s}^{(1)}, \underline{s}^{(2)}, \cdots, \underline{s}^{(N-1)})$. The sequence $\underline{c}$ is referred to as a PPCSS if the periodic autocorrelation functions of all its sub-sequences sum to zero apart from time shift $\tau = 0$, namely,

$$R_{c,c}(\tau) = \sum_{i=0}^{N-1} R_{s^{(i)},s^{(i)}}(\tau) = \begin{cases} NM & \tau \equiv 0 \pmod{M} \\ 0 & \tau \not\equiv 0 \pmod{M}. \end{cases} \tag{2}$$

3 New Construction of PPCSSs

In this subsection, a new construction for PPCSSs is presented. For arbitrary given positive integers N and M , we freely choose a PPS $\underline{a}$ with length $L = MN$ expressed by

$$\underline{a} = (a_{\lambda}, a_{\lambda+1}, \cdots, a_{\lambda+N-1}, a_{\lambda+N}, a_{\lambda+N+1}, \cdots, a_{\lambda+2N-1}, \cdots, a_{\lambda+(M-1)N}, a_{\lambda+(M-1)N+1}, \cdots, a_{\lambda+(M-1)N+N-1}), \quad (3)$$

where $0 \le \lambda \le L-1$, i.e., we consider the PPS in arbitrary a period.

Now, we carry out sampling the PPS $\underline{a}$ in (3) with equal space $N-1$. More clearly, we assign an element with subscript $\lambda + l \ (0 \le l \le N-1)$ in (3) as an initial one, and extract those element whose subscripts are exactly $\lambda + kN + l \ (0 \le k \le M-1)$. Hence, by arranging all obtained elements in natural subscripts' order, a sub-sequence is yielded, which is denoted by $\underline{b}^{(\lambda+l)_N}$ for convenience, where $(x)_N$ denotes the residue of x modulo N. After l ranges in the range from 0 to $N-1$, a PPCSS is obtained, which consists of all the obtained sub-sequences. For the sake of understanding easy to the reader, the operation referred to above is visually described in Fig. 1. In a mathematical term, we have the following N sub-sequences $\underline{b}^{(\lambda+l)_N} \ (0 \le l \le N-1)$.

$$\begin{aligned}
\underline{b}^{(\lambda)_N} &= (b_0^{(\lambda)_N}, b_1^{(\lambda)_N}, b_2^{(\lambda)_N}, \cdots, b_{M-1}^{(\lambda)_N}) \\
\underline{b}^{(\lambda+1)_N} &= (b_0^{(\lambda+1)_N}, b_1^{(\lambda+1)_N}, b_2^{(\lambda+1)_N}, \cdots, b_{M-1}^{(\lambda+1)_N}) \\
&\vdots \\
\underline{b}^{(\lambda+N-1)_N} &= (b_0^{(\lambda+N-1)_N}, b_1^{(\lambda+N-1)_N}, b_2^{(\lambda+N-1)_N}, \cdots, b_{M-1}^{(\lambda+N-1)_N}),
\end{aligned} \quad (4)$$

where $b_k^{(\lambda+i)_N} = a_{\lambda+kN+i} \ (i = 0,1,2,\cdots,N-1; k = 0,1,2,\cdots,M-1)$.

Note that the indices $(\lambda+i)_N \ (i = 0,1,2,\cdots,N-1)$ of sub-sequences in (4) are given only for the sake of convenient expression in Fig.1, in fact, those indices can be freely assigned.

Then we declare the following conclusion.

Theorem 1: The sequence $\underline{c} = (\underline{b}^{(\lambda)_N}, \underline{b}^{(\lambda+1)_N}, \underline{b}^{(\lambda+2)_N}, \cdots, \underline{b}^{(\lambda+N-1)_N})$ is a PPCSS.

Proof: We consider the summation of the periodic autocorrelation functions of all sub-sequences in (4), therefore we have

$$\begin{aligned}
R_{c,c}(\tau) &= \sum_{i=0}^{N-1} R_{b^{(\lambda+i)_N}, b^{(\lambda+i)_N}}(\tau) \quad (|\tau| \le M-1) \\
&= \sum_{i=0}^{N-1} \sum_{d=0}^{M-1} b_d^{(\lambda+i)_N} [b_{d+\tau}^{(\lambda+i)_N}]^*
\end{aligned}$$

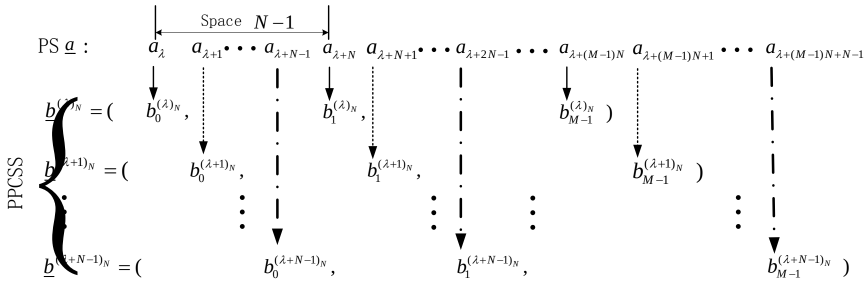

Fig. 1. A description on new construction for PPCSSs

$$= \sum_{i=0}^{N-1} \sum_{d=0}^{M-1} a_{\lambda+dN+i}[a_{\lambda+(d+\tau)N+i}]^* \quad (5)$$

$$= \sum_{k=0}^{L-1} a_{k+\lambda}(a_{k+\lambda+\tau N})^*$$

$$= R_{a,a}(\tau N).$$

Note that the sequence $\underline{a}$ in (3), employed by us, is a PPS, therefore its periodic autocorrelation function satisfies

$$R_{a,a}(u) = \sum_{k=0}^{L-1} a_{k+\lambda}(a_{k+\lambda+u})^* = \begin{cases} L & u \equiv 0 \pmod{L} \\ 0 & u \not\equiv 0 \pmod{L}. \end{cases} \quad (6)$$

Hence, due to $|\tau N| \le L-1\ (|\tau| \le M-1)$ and in accordance with (5) and (6), we obtain

$$R_{c,c}(\tau) = \begin{cases} L & \tau \equiv 0 \pmod{M} \\ 0 & \tau \not\equiv 0 \pmod{M}, \end{cases} \quad (7)$$

which illuminates that the theorem is true. We are done. Q.E.D.

For a combination number L, it is possible that there are several factors. For instance, $L = 12 = 2 \times 6 = 3 \times 4$. Apparently, for a given PPS $\underline{a}$ with a period L of combination number, each kind of decomposition of L will results in a different PPCSS. For example, when $L = 12$, from Theorem 1 there are four classes of the resulting PPCSSs with $(M, N) = (2,6)$, $(3,4)$, $(4,3)$, and $(6,2)$, respectively. Hence, we refer to all the resultant PPCSSs as a class of PPCSSs associated with $\underline{a}$. This motivates that we investigate how many classes of PPCSSs Theorem 1 can produce for a given PPS. For solving this problem, a following lemma is necessary.

Lemma 2: [10] Let $A(L,P)$ be the total number of PPSs with length L and alphabet size P . Let $L = sm^2$, where s and m are both positive integers with s square-free. Then

$$A(L,P) \geq \begin{cases} m!s^m \phi^m(s) P^m & P_{\min} \text{ divides } P \\ 0 & \text{otherwise}, \end{cases} \quad \mathbf{(8)}$$

Where

$$P_{\min} = \begin{cases} 2sm & \text{for } s \text{ even and } m \text{ odd} \\ sm & \text{otherwise}, \end{cases} \quad \mathbf{(9)}$$

and $\phi(g)$ denotes Euler function [5].

The following theorem answers our problem.

Theorem 2: Let the conditions be the same ones in Lemma 2. Then, Theorem 1 results in $A(L,P)$ classes of PPCSSs.

According to Lemma 2 and Table 3.1 in [11], from Theorem 1 the number of classes of the resultant PPCSSs up to length $L = 12$ and $P = 10$ is given in Table 1.

Table 1. Number of classes of PPCSSs from Theorem 1 up to $L = 12$ and $P = 10$.

L \ P	2	3	4	5	6	7	8	9	10	PPCSSs (M,N)'s
2			8				16			
3		8			36			54		
4	8		32		72		128		200	(2,2)
5				100					200	
6										(2,3),(3,2)
7						294				
8			128				512			(2,4),(4,2)
9		162			1296			4374		(3,3)
10										
11										
12					2592					(2,6),(3,4),(4,3),(6,2)

4 Conclusions

In this paper, a new construction which can produces PPCSSs is presented, and in the resulting PPCSSs, the number and length of sub-sequences can be changed on demand. In addition, the family size of the proposed PPCSSs is equivalent to that of the employed PPSs. The advantage mentioned above highlights that new construction in this paper is very fit for various engineering applications. Incidentally, Theorem 1 can be applied to any perfect sequences, such as ternary perfect sequences[13], multilevel perfect sequences[14], QPSK+ perfect sequences[15], 8-QAM+ perfect sequences[16], and so on, so as to produce periodic ternary complementary sequence sets (CSSs), periodic multilevel CSSs, periodic QPSK+ CSSs, periodic 8-QAM+ CSSs, etc..

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