

## Research on Complex Networks Control Based on Fuzzy Integral Sliding Theory

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**Abstract.** A fuzzy integral sliding mode control method based on the finite time stable theory and integral sliding mode control theory is proposed to solve a class of complex network problems. The complex networks are expressed by T-S fuzzy models with bounded approximation errors by using the approximation capability of T-S fuzzy models. The paper proposes fuzzy dynamic integral sliding mode control scheme and the scheme is designed for the complex networks based on their T-S fuzzy approximation models.

**Keywords:** Fuzzy, complex networks, integral sliding mode, nonlinear system..

### 1 Introduction

Sliding mode control is also called variable structure control. It is under the action of the sliding mode controller, and controlled system comes into high frequency switched state to force it move continuously on the sliding surface. Eventually it will converge to the desired sliding mode along the sliding mode surface so as to achieve the goal of control. Sliding mode control is a very effective method in dealing with uncertain high-order nonlinear dynamic system [1]. It gives expression to the uncertain nonlinear system's smaller error, external disturbance's strong robustness and the simple algorithm design. But it can produce high frequency chattering of sliding mode in the actual sliding mode control system due to the influence of the non-ideal effects. With the development of complex networks in recent years, complex network appears in various kinds of actual system. A complex network composes of many nodes and every node connects with each other. It is high dimension, strong coupling and uncertainty, so the traditional single node control theory is difficult to solve multiple nodes complex network system's analysis and design problems. In the past ten years, many researches are based on the analysis of the T - S fuzzy model in order to study the issue of complex nonlinear systems and complex networks. A series of fuzzy integral sliding mode control scheme of the nonlinear system can be put forward with this model, because it provides the advantage of traditional control theory and technology.

This paper puts forward a new kind of fuzzy dynamic integral sliding mode control method to realize the stability of the nonlinear system. The key is that the integral sliding surface depends on both of the system state vector  $x$  and control input vector  $u$ . The sliding motion can be guaranteed by solving LMI matrix, and the integral sliding surface along with the dynamic sliding-mode controller [2] can be obtained.

## 2 Problem descriptions and the preliminary knowledge

The  $i$ th child node of the complex network can be expressed in formula (1) :

$$dx_i(t) = f(x_i(t), u_i(t)) \quad (1)$$

Consider a class of complex network composed of  $N$  child nodes  $S_i$ ,  $i = 1, 2, \dots, N$ . Each rule of child node  $S_i$  can use T - S fuzzy model to describe:

If  $\zeta_{i1}$  is  $M_{i1}^l$  and ...and  $\zeta_{ig_i}$  is  $M_{ig_i}^l$ ; Then

$$dx_i(t) = F_i^l x_i(t) + G_{1i}^l w_i(t) + G_{2i}^l u_i(t) + \Omega_i(t) \quad (2)$$

$$\Omega_i(t) = \sum_{j=1, j \neq i}^N H_{ji} x_j(t) \quad (3)$$

$$z_i(t) = K_i^l x_i(t) + L_i u_i(t) \quad (4)$$

$$y_i(t) = K_{yi}^l x_i(t) + L_{yi}^l w_i(t), i = 1, 2, \dots, N, l = 1, 2, \dots, r_i \quad (5)$$

$F_i^l$ ,  $G_{1i}^l$ ,  $G_{2i}^l$ ,  $K_i^l$ ,  $L_i$  represent the rule node matrix of the  $i$ th child node.  $H_{ji}$  represents the interconnection matrix of the  $i$ th child node and the  $j$ th child node.  $x_i \in R^{n_i}$ ,  $u_i \in R^{m_i}$ ,  $w_i$ ,  $z_i$ ,  $y_i$  respectively represent the system state, control, disturbance, measurable output and the output vector. If adopting the single point blurred, the product of reasoning and center average solution of the blur, the above formula can be represented as:

$$dx_i(t) = \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t)) (F_i^l x_i(t) + G_{1i}^l w_i(t) + G_{2i}^l u_i(t) + \sum_{j=1, j \neq i}^N H_{ji} x_j(t)) \quad (6)$$

$$z_i(t) = \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t)) (K_i^l x_i(t) + L_i u_i(t)) \quad (7)$$

$$y_i(t) = \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t)) (K_{yi}^l x_i(t) + L_{yi}^l w_i(t)) \quad (8)$$

$$\gamma_i^l(\zeta_{iq}(t)) = \prod_{q=1}^{g_i} M_{iq}^l(\zeta_{iq}(t)), \quad \eta_i^l(\zeta_{iq}(t)) = \frac{\gamma_i^l(\zeta_{iq}(t))}{\sum_{l=1}^{r_i} \gamma_i^l(\zeta_{iq}(t))} \quad (9)$$

$\zeta_{i1}, \dots, \zeta_{ig_i}$  is antecedent variable,  $M_{iq}^l(\zeta_{iq}(t))$  is membership of  $M_{iq}^l$ . It is observed that  $\gamma_i^l(\zeta_{iq}(t)) \geq 0$ ,  $\eta_i^l(\zeta_{iq}(t)) \geq 0$ ,  $\sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t)) = 1, i=1, \dots, J, l=1, \dots, r_i$ ,  $r_i$  is  $i$ th child node rule's amount. For any complex network,  $f(x_i(t), u_i(t))$  is in the interval of  $X \times U$  and  $f(0,0) = 0$ . There exist a positive constant  $\varepsilon_f$ . So the T-S fuzzy model is described as follows:

$$\hat{f}(x_i, u_i) = \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t))(F_i^l x_i(t) + G_{1i}^l w_i(t) + G_{2i}^l u_i(t) + \sum_{j=1, j \neq i}^N H_{ji} x_j(t)) \quad (10)$$

$$f(x_i, u_i) = \hat{f}(x_i, u_i) + \Delta E(x_i, u_i) \\ = \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t))((F_i^l + \Delta F_i^l)x_i(t) + G_{1i}^l w_i(t) + (G_{2i}^l + \Delta G_{2i}^l)u_i(t) + \sum_{j=1, j \neq i}^N H_{ji} x_j) \quad (11)$$

$$\|\Delta E(x, u)\| = \left\| \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t))[\Delta F_i^l, \Delta G_{2i}^l] * [X^T, U^T]^T \right\| \quad (12)$$

$$\|\Delta F_i^l, \Delta G_{2i}^l\| \leq \varepsilon_f \quad (13)$$

According to (6), (9), (11), the approximation error upper bound value can obtain small enough by selecting enough large values of fuzzy rules in the T-S fuzzy model[3]. Through the analysis of the approximation error, one can get:

$$\dot{x}_i(t) = \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t))((F_i^l + \Delta F_i^l)x_i(t) + G_{1i}^l w_i(t) + (G_{2i}^l + \Delta G_{2i}^l)u_i(t) + \sum_{j=1, j \neq i}^N H_{ji} x_j) \quad (24)$$

### 3 The design of dynamic integral sliding mode control

For the provided nonlinear model(1), (6), the following fuzzy integral sliding surface is designed as follows:

$$\begin{cases} S_i(t) = A_i^x d_i(t) + A_i^u e_i(t) - \int_0^t \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t)) A_i^x (F_i^l x_i(\phi) + G_{2i}^l u_i(\phi)) d\phi - \int_0^t \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t)) A_i^u ( \\ d_i(t) = x_i(t) - x_i(0) \\ e_i(t) = u_i(t) - u_i(0) \end{cases} \quad (35)$$

$A_i^x$ 、 $A_i^u$ 、 $E_i^l$ 、 $H_i^l$  respectively describe  $m^*n$ 、 $m^*m$ 、 $m^*n$ 、 $m^*m$  matrix.

Theorem: Consider the nonlinear system in formula (1) and the fuzzy system in formula (6). One can use the following fuzzy controller:

If  $\zeta_{i1}$  is  $M_{i1}^l$  and ... and  $\zeta_{ig_i}$  is  $M_{ig_i}^l$ ; Then

$$\dot{u}_i(t) = E_i^l x_i(t) + H_i^l u_i(t) - G_{ii}^l w_i(t) A_i^{u^1} A_i^x - \sum_{j=1, j \neq i}^N H_{ji} x_j(t) A_i^{u^1} A_i^x - (\omega + v_i(t)) A_i^{u^1} \text{sgn}(s_i(t)), i=1, \dots, J, l=1, \dots, r_i \quad (46)$$

the fuzzy controller using fuzzy rules can also be described as follows:

$$\dot{u}_i(t) = \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t)) \left[ E_i^l x_i(t) + H_i^l u_i(t) - G_{ii}^l w_i(t) A_i^{u^1} A_i^x - \sum_{j=1, j \neq i}^N H_{ji} x_j(t) A_i^{u^1} A_i^x \right] - (\omega + v_i(t)) A_i^{u^1} \text{sgn}(s_i(t)) \quad (57)$$

$$v_i(t) = \varepsilon_f \left\| A_i^x \right\| \left\| [x_i^T(t), u_i^T(t)]^T \right\| \quad (68)$$

$\omega$  is a positive constant, the closed-loop control system trajectory can keep on the sliding surface almost since the finite time by using the above integral controller, Furthermore constructing Lyapunov function[4] for the subsystem function to prove:

$$V(t) = \frac{1}{2} \sum_{i=1}^N s_i(t) s_i^T(t) \quad (79)$$

$$\dot{s}_i(t) = A_i^x \dot{x}_i(t) + A_i^u \dot{u}_i(t) - \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t)) A_i^x (F_i^l x_i(t) + G_{2i}^l u_i(t)) dt - \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t)) A_i^u (E_i^l x_i(t) + H_i^l u_i(t)) dt \quad (20)$$

Then from(14), (17), (20)one has:

$$V(t) = \frac{1}{2} \sum_{i=1}^N s_i(t) s_i^T(t) \quad (21)$$

$$\dot{s}_i(t) = A_i^x \dot{x}_i(t) + A_i^u \dot{u}_i(t) - \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t)) A_i^x (F_i^l x_i(t) + G_{2i}^l u_i(t)) dt - \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t)) A_i^u (E_i^l x_i(t) + H_i^l u_i(t)) dt \quad (22)$$

Then from (18), (19), (22)one has:

$$\dot{V}(t) \leq -\sum_{i=1}^N \omega \|s_i(t)\| = -\sum_{i=1}^N \omega \sqrt{2V(t)} \quad (23)$$

Through the above formula one can prove that the system's trajectory[5] can keep on the sliding surface in the limited time, and the above argument has given the relevant proof. So define other sliding surface equation in order to describe the trajectory conveniently.

$$s_i(t) = \bar{A}\{\bar{f}_i(t) - \int_0^t \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t))(C_i^1 \bar{F}_i^l + C_i^2 \bar{E}_i^l) \bar{x}_i(\phi) d\phi\} \quad (24)$$

$$\bar{f}_i(t) = \bar{x}_i(t) - \bar{x}_i(0), \quad \bar{x}_i(t) = [x_i^T(t), u_i^T(t)]^T, \quad C_i^1 = [I_n, 0_{n \times m}]^T, \quad C_i^2 = [0_{m \times n}, I_m]^T, \\ \bar{F}_i^l = [F_i^l, G_{2i}^l], \quad \bar{A} = [A_i^x, A_i^u], \quad \bar{E}_i^l = [E_i^l, H_i^l]$$

One can draw the following two closed loop control system equation:

$$\dot{x}_i(t) = \hat{f}(x_i, u_i) = \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t))(F_i^l x_i(t) + G_{1i}^l w_i(t) + G_{2i}^l u_i(t) + \sum_{j=1, j \neq i}^N H_{ji} x_j(t)) \quad (25)$$

$$\dot{u}_i(t) = \hat{g}(x_i, u_i) = \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t))(E_i^l x_i(t) + G_{1i}^l w_i(t) + H_i^l u_i(t) + \sum_{j=1, j \neq i}^N H_{ji} x_j(t)) \quad (26)$$

The above two equations are equivalent to:

$$\dot{\bar{x}}_i(t) = \sum_{l=1}^{r_i} \eta_i^l(\zeta_{iq}(t))[(C_i^1 \bar{F}_i^l + C_i^2 \bar{E}_i^l) \bar{x}_i(t) + \sum_{j=1, j \neq i}^N H_{ji} x_j(t) + G_{1i}^l w_i(t)] \quad (27)$$

If the design of closed-loop control equation (27) is reasonable, then the stability of sliding mode controller can be guaranteed.

## 4 Conclusion

A novel dynamic integral sliding mode control scheme has been developed for complex networks systems based on T-S fuzzy models. It has been shown that the sliding mode can be achieved and maintained almost surely since the initial time with the integral sliding surface and a novel dynamic sliding-mode controller. And sufficient conditions to guarantee the stability of the sliding motion are given in terms of LMIs.

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