Advanced Science and Technology Letters
Vol.83 (ISA 2015), pp.47-51
http://dx.doi.org/10.14257/astl.2015.83.10

A SMC method for A Class of Nonlinear Systems based on SVD technique

Dan Zhao[1], Hongyu Zhang[2], Yue Han[3], Zisong Xiao[4]

[1]Department of Fundamental Teaching, Shenyang Institute of Engineering, Shenyang
[2]State Grid Liaoning Electric Power Supply Co., LTD., Liaoning Province, China
[3]Electric Power Research Institute, State Grid Liaoning Electric Power Supply Co. Ltd, Shenyang, China
[4]Liaoning State Grid Electric Power Transmission & Transformation Engineering Company
zhaodan@sie.edu.cn

Abstract. The sliding mode control problem is investigated for a class of nonlinear systems with the unmatched nonlinearity which satisfies a quadratic constraint. Based on Lyapunov stability theory, utilizing the singular value decomposition method, the sliding mode control is formulated to stabilize the considered systems.

Keywords: Sliding mode control, unmatched condition, tuned parameters, bound on the nonlinearity.

1 Introduction

Since sliding mode control (SMC) possesses the strong robustness ability to eliminate or compensate the model uncertainties, parameter variations, and external disturbances [1-5], SMC is an effective control approach and is extensively researched. In [6], a novel SMC law is proposed to deliver the tip of a flexible asymmetric-tipped needle to a desired point or to track a desired trajectory within tissue. In [7], an approach known as the TSMC is employed to achieve the objective of fast converging times without excessive control effort.

However, there are few reports on optimization for control gain and upper bound of the unmatched nonlinearity, which is the motivation of the present paper. In this paper, two state transformation techniques, which are singular value decomposition transformation and descriptor system model transformation, are employed to obtain the existence condition of sliding surface and the upper bound of the unmatched nonlinear term. Also, as a key characteristic of this paper, the tuned parameters are introduced to reduce the chattering phenomenon. Under the solvable condition of the convex problem, one can obtain the upper bound on the nonlinear term which guarantees the considered systems are stable.

The problem formulation for the considered system based on sliding mode control law is described in Section 2. The sliding surface design and the reaching motion control design are developed in Section 3. In Section 4, a numerical example is given to illustrate the proposed method. Finally, one can obtain some conclusions in Section

ISSN: 2287-1233 ASTL

5.

2 Problem formulation

Consider the following nonlinear systems

$$\begin{aligned} \dot{x}(t) &= [A + \Delta A(t)]x(t) + [A_\sigma + \Delta A_\sigma(t)]x(t-\sigma(t)) + Bu(t) + h(x) \\ x(t) &= \phi(t), \qquad t \in [-f_0, 0] \end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ is the state of the system, and $u(t) \in \Re^m$ is the input vector. $A \in \Re^{n\times n}$, $A_\sigma \in \Re^{n\times n}$, and $B \in \Re^{n\times m}$ are known constant matrices, and $h:\Re^n \to \Re^n$ represents a piecewise-continuous nonlinear function satisfying $h(0)=0$. It is assumed that the nonlinear term $h(x)$ can be bounded by a quadratic inequality

$$h^{\mathrm{T}}(x)h(x) \le \alpha^2 x^{\mathrm{T}} H^{\mathrm{T}} H x \tag{2}$$

where H is a constant matrix, and $\alpha > 0$ is a scalar parameter. $\phi(t)$ is the initial condition. $\sigma(t)$ is the time-varying delays. Assume that there exist constants f_0, and f satisfying

$$0 \le \sigma(t) \le f_0, \qquad \dot{\sigma}(t) \le f < 1. \tag{3}$$

Time-varying parametric uncertainties $\Delta A(t)$ and $\Delta A_\sigma(t)$ are assumed to be of the following form

$$\Delta A(t) = EF(t)D, \quad \Delta A_\sigma(t) = E_\sigma F_\sigma(t) D_\sigma$$

where E, D, E_σ and D_σ are constant matrices of appropriate dimensions, and $F(t)$ and $F_\sigma(t)$ are the unknown matrix function satisfying

$$F^{\mathrm{T}}(t)F(t) \le I,\ F_\sigma^{\mathrm{T}}(t)F_\sigma(t) \le I,\ \forall t \ge 0.$$

In this paper, we assume that $rank(B) = m$. One can easily get the singular value decomposition of B:

$$B = [U_1 \quad U_2]\begin{bmatrix} \Sigma \\ 0_{(n-m)\times m} \end{bmatrix} V^{\mathrm{T}} \tag{5}$$

where $\Sigma \in \Re^{m\times m}$ is a diagonal positive-definite matrix, $U_1 \in \Re^{n\times m}$, $U_2 \in \Re^{n\times(n-m)}$ and $V \in \Re^{m\times m}$ are unitary matrices.

Choose $z = \Gamma x$, where $\Gamma^T = [U_1 \quad U_2]$, we can obtain another form of (1) as follows:

$$
\begin{aligned}
\dot{z}_1(t) &= \left[\bar{A}_{11} + \Delta\bar{A}_{11}(t)\right] z_1(t) + \left[\bar{A}_{12} + \Delta\bar{A}_{12}(t)\right] z_2(t) + \left[\bar{A}_{\sigma 11} + \Delta\bar{A}_{\sigma 11}(t)\right] z_1(t-\sigma(t)) \\
&\quad + \left[\bar{A}_{\sigma 12} + \Delta\bar{A}_{\sigma 12}(t)\right] z_2(t-\sigma(t)) + B_1 u(t) + \bar{h}_1(\Gamma^{-1} z) \\
\dot{z}_2(t) &= \left[\bar{A}_{21} + \Delta\bar{A}_{21}(t)\right] z_1(t) + \left[\bar{A}_{22} + \Delta\bar{A}_{22}(t)\right] z_2(t) + \left[\bar{A}_{\sigma 21} + \Delta\bar{A}_{\sigma 21}(t)\right] z_1(t-\sigma(t)) \\
&\quad + \left[\bar{A}_{\sigma 22} + \Delta\bar{A}_{\sigma 22}(t)\right] z_2(t-\sigma(t)) + \bar{h}_2(\Gamma^{-1} z) \\
z_1(t) &= \bar{\phi}_1(t), \qquad t \in [-f_0, 0] \\
z_2(t) &= \bar{\phi}_2(t), \qquad t \in [-f_0, 0]
\end{aligned} \tag{6}
$$

where $z_1(t) \in \Re^{m}$, $z_2(t) \in \Re^{n-m}$, $\bar{A}_{11} = U_1^{\mathrm{T}} A U_1$, $\bar{A}_{12} = U_1^{\mathrm{T}} A U_2$, $\bar{A}_{21} = U_2^{\mathrm{T}} A U_1$, $\bar{A}_{22} = U_2^{\mathrm{T}} A U_2$, $\bar{h}_1(\Gamma^{-1} z) = U_1^{\mathrm{T}} h(\Gamma^{-1} z)$, $\bar{h}_2(\Gamma^{-1} z) = U_2^{\mathrm{T}} h(\Gamma^{-1} z)$.

We construct a sliding surface as follows:

$$
S(t) = \left[I \quad C\right] z(t) = z_1(t) + C z_2(t) = 0 \tag{7}
$$

Noting the second equation of (6) and (7), we can obtain the following sliding motion:

$$
\begin{aligned}
\dot{z}_2(t) &= \left[\bar{A}_{22} - \bar{A}_{21}C + \Delta\bar{A}_{22}(t) + \Delta\bar{A}_{21}(t)C\right] z_2(t) + \bar{h}_2(\Gamma^{-1} z) \\
&\quad + \left[\bar{A}_{\sigma 22} - \bar{A}_{\sigma 21}C + \Delta\bar{A}_{\sigma 22}(t) + \Delta\bar{A}_{\sigma 21}(t)C\right] z_2(t-\sigma(t)) \\
z_2(t) &= \bar{\phi}_2(t), \qquad t \in [-f_0, 0]
\end{aligned} \tag{8}
$$

where $\bar{h}_2^{\mathrm{T}}(\Gamma^{-1} z)\bar{h}_2(\Gamma^{-1} z) \le \alpha^2 z_2^{\mathrm{T}}(t)\left[-C^{\mathrm{T}} \quad I\right]\bar{H}^{\mathrm{T}}\bar{H}\begin{bmatrix} -C \\ I \end{bmatrix} z_2(t)$, and $\bar{H} = H\Gamma^{-1}$.

3 Main result

Theorem The sliding motion (8) is quadratically stable with upper bound α^*, if the following optimization problem

$$
\min \quad \gamma + K_X + K_{Q_2} \tag{9}
$$

subject to

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} & 0 & \Omega_{18} \\ * & \Omega_{22} & \Omega_{23} & \Omega_{24} & 0 & \Omega_{26} & 0 & 0 \\ * & * & \Omega_{33} & 0 & 0 & 0 & \Omega_{37} & 0 \\ * & * & * & \Omega_{44} & 0 & 0 & 0 & 0 \\ * & * & * & * & \Omega_{55} & 0 & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 & 0 \\ * & * & * & * & * & * & \Omega_{77} & 0 \\ * & * & * & * & * & * & * & \Omega_{88} \end{bmatrix} < 0 \tag{10}$$

$$\gamma - \frac{1}{\bar{\alpha}^{2}} < 0 \tag{11}$$

$$\begin{bmatrix} -K_X I & X^{\mathrm{T}} \\ * & -I \end{bmatrix} < 0 \tag{12}$$

$$\begin{bmatrix} Q_2 & I \\ * & K_{Q_2} I \end{bmatrix} > 0 \tag{13}$$

has a solution set (positive scalars $\gamma, \varepsilon_1, \varepsilon_2, K_{Q_2}, K_X$, positive definite matrices Q_1, Q_2, Q_3, and matrix X). Furthermore, the gain C of the sliding surface (7) and the upper bound α^* are obtained as follows

$$C = XQ_2^{-1}, \qquad \alpha^* = \sqrt{\frac{1}{\gamma}}$$

where $\bar{\alpha}$ is a given scalar parameter in (2), and

$\Omega_{11} = \bar{A}_{22}Q_2 - \bar{A}_{21}X + Q_2\bar{A}_{22}^{\mathrm{T}} - X^{\mathrm{T}}\bar{A}_{21}^{\mathrm{T}} + Q_3$, $\Omega_{12} = Q_1 - Q_2 + Q_2\bar{A}_{22}^{\mathrm{T}} - X^{\mathrm{T}}\bar{A}_{21}^{\mathrm{T}}$,

$\Omega_{13} = \bar{A}_{\sigma 22}Q_2 - \bar{A}_{\sigma 21}X$, $\Omega_{14} = \varepsilon_1 U_2^{\mathrm{T}} E$, $\Omega_{15} = Q_2 U_2^{\mathrm{T}} D^{\mathrm{T}} - X^{\mathrm{T}} U_1^{\mathrm{T}} D^{\mathrm{T}}$, $\Omega_{16} = \varepsilon_2 U_2^{\mathrm{T}} E_\sigma$,

$\Omega_{18} = \left[-X^{\mathrm{T}} \quad Q_2 \right] \bar{H}^{\mathrm{T}}$,

$\Omega_{22} = -Q_2 - Q_2 + I$, $\Omega_{23} = \Omega_{13}$, $\Omega_{24} = \Omega_{14}$, $\Omega_{26} = \Omega_{16}$,

$\Omega_{33} = -(1 - f)Q_3$, $\Omega_{37} = Q_2 U_2^{\mathrm{T}} D_{\sigma}^{\mathrm{T}} - X^{\mathrm{T}} U_1^{\mathrm{T}} D_{\sigma}^{\mathrm{T}}$, $\Omega_{44} = -\varepsilon_1 I$,

$\Omega_{55} = -\varepsilon_1 I$, $\Omega_{66} = -\varepsilon_2 I$,

$\Omega_{77} = -\varepsilon_2 I$, $\Omega_{88} = -\gamma I$.

4 Conclusion

The sliding mode control problem of nonlinear systems with unmatched nonlinearity is complex and challenging. Based on f the Lyapunov stability theory, employing the singular- value-decomposition algorithm and descriptor-system-model-transformation method, the sliding surface and the reaching control law are designed, which can be obtained easily by solving the according optimization problem.

In this paper, based on green data classification strategy based on anticipation (AGDC) the data will be divided into five category, they are new data、old data、hot data, cold data, seasonal data.

References

1. Pang, H. and Tang, G.: ICIC Exp. Lett.. 4, 6(2010).
2. Xia, Y., Zhu, Z., and Fu, M.: IET Control Theory Appl., 5, 1 (2011).
3. Yan, M. and Shi, Y.: IET Control Theory Appl., 2, 8 (2008).
4. Feng, Y., Han, F. L., Yu, X. H.: Automatica. 50, 4 (2014).
5. Temal, T., Ashrafiuon, H.: Electron Lett. 48, 15 (2012)
6. Rucker, D. C., Das, J., Hunter, H. B., Swaney, P. J., Miga, M. I., Sarkar, N., Webster, R. J.: IEEE T Robot. 29, 5 (2013).
7. Ghasemi, M., Nersesov, S. G., Clayton, G., Franklin, J: I. 351, 5 (2014).