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Optimal Decisions and Coordination Mechanisms for a Fresh Agricultural Product Supply Chain System

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Abstract

For a one-supplier-one-retailer agricultural product supply chain, quantity and quality of the agricultural products often reduce because their own perishable and anthropogenic factors in the process from the supplier via the retailer to final consumers. This paper takes shortage and deterioration factors into account, establishes related profit functions of the supplier and the retailer, obtains the optimal ordering cycle and quantity decisions, and analyzes the impact of deterioration on decision makers under the decentralized system. To maximize the total profit of supply chain, we explore coordination mechanisms for the supply chain when considering shortage and deterioration factors, such as revenue sharing contract, quantity discount contract and buy-back contract. By comparing these different contracts, the coordination effect is analyzed in details. Finally, through series of numerical analysis, some managerial insights are provided.

Keywords: *fresh agricultural product; shortage rate; coordination; revenue sharing contract*

1. Introduction

As the level of income is significantly improved, Chinese supermarket retail industry has been developing rapidly. In the middle of 90s, chain supermarket begins to set foot in fresh agricultural products business while the fresh agricultural product market has become a hot contested spot between supermarkets and farmers' markets. Agricultural products entering supermarkets, to some extent, brings convenience and food safety, but also derives a series of problems. Too many intermediaries in supermarkets weaken agricultural products as well as freshness directly. According to the relevant statistics, the rate of agricultural products circulation losses generally remains 1.7%~5.0% in developed countries, while in China the rate of fruit and vegetable losses comes up to 20%~30%. Thus, losses of agricultural products and a lack of coordination in circulation of agricultural products supply chain have become key factors of restricting the development of Chinese agricultural products. Therefore, according to characteristics of metamorphic losses of agricultural products, how to develop reasonable circulation modes of cooperation and ordering policies is on the front burner.

A fresh agricultural product supply chain refers to a supply chain about the fresh agricultural products, such as fruits and vegetables, dairy products, poultry meat, aquatic products, and so on. Fresh agricultural products are perishable, which, in the whole process of circulation, even can deteriorate due to the long circulation because of freshness reduction, human behavior or other exterior factors. These characteristics make fresh agricultural products distinguished from the general products. Therefore, comprehensively considering the impact of such characteristics is pretty necessary in the course of researching on fresh agricultural products. The study is to explore the optimal

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ordering policies of fresh agricultural products, and not only is the impact of deterioration on supply chain taken into account, but also the effective of contracts on supply chain coordination, see Chen and Xiao (2011) [1].

This article is mainly related to inventory policies of deteriorating items which have been concerned by many researchers. For example, taking the time discount into account, Chung and Lin (2001) examine the consumption products inventory replenishment models [2]. Khanra and Chaudhuri (2003) assume that demand is the quadratic function of time set up inventory model according to two types of situations, including the finite and infinite planning periods [3]. Law and Wee (2006) make the establishment of a joint production inventory model under the discount on the assumptions that rate of value-added products consistent with the two-parameter Weibull distribution function is a decreasing function of the time, deterioration rate is an increasing function of time [4]. Ferguson and Koenigsberg (2007) propose a two-stage model, namely, as the quality of the remaining inventory is lower in the eyes of the customer, enterprises can thus decide whether to prepare or not for surplus stocks to the next stage [5]. In the context of supply chain management, Lin and Lin (2007) establish two levels of jointly managed inventory model, where there is no limitation that qualified orders have to be equal to the cycle, and complete buy-backs are allowed [6]. Lee and Dye (2012) formulate a deteriorating inventory model with stock-dependent demand by allowing preservation technology cost as a decision variable in conjunction with replacement policy [7]. More perishable inventory research can refer to Whitin (1957) [8], Goyal and Giri (2001) [9], Blackburn and Scudder (2009) [10].

For the research of fresh product supply chain, Ferguson and Ketzenberg (2006) study on the use of information sharing in the supply chain to improve the freshness of perishable retail products [11]. Cai *et. al.*, (2009) develop a fresh product supply chains model with freshness-keeping effort to characterize each party's optimal decisions in both decentralized and centralized systems, and they develop an incentive scheme to facilitate coordination between the two parties [12]. Chung *et. al.*, (2012) develop a model to simultaneously determine the inventory and pricing policies of a retailer with contractual agreement, in which they provide a comprehensive analysis to derive the optimal special sale period for deteriorating item [13].

In terms of contracts for supply chain coordination, a lot of work has been done. Taylor (2002) puts forward the idea that it is feasible to use supply chain channel rebate to make sales efforts so as to coordinate the entire supply chain profits [14]. Cachon (2003) provides an overview of supply chain coordination with contract [15]. Song and Saibal (2008) study the structural buy-back contract for price makers—retailers, and also explore this buy-back contract on the coordination of supply chain [16]. Seifert *et. al.*, (2012) explicitly analyze the order quantity and contracting decisions for a decentralized three-echelon supply chain, and compare the supply chain efficiency when there is upstream coordination and when there is downstream coordination and show that the former is more efficient than or as efficient as the latter [17]. More traditional contracts can be found from Parlar and Wang (1994) [18], Eppen and Iyer (1997) [19], Tsay (1999) [20].

The existing studies focus more on issues of deteriorating products about the inventory control, algorithm design and information sharing, *etc.*, but rarely considers the coordination issue with deterioration and shortage rate. Taking fresh agricultural product as the research object, this article formulates the supplier's and the retailer's profit functions, and the contribution is threefold. The first is that the optimal ordering policies are obtained including both shortage and deterioration of fresh agricultural products. The second is that three coordinating contracts are provided, and the impacts of shortage and deterioration on contracts, optimal decisions and supply chain performance are analyzed. Finally, through numerical analysis, we show that shortage and deterioration have significant impact on the supply chain, and the effects with different contracts are different. This research is helpful in improving the operation performance of managers in the fresh agricultural product supply chain.

2. Model Description

Consider a two-echelon supply chain consist of a supplier and a retailer where the powerful retailer has to make ordering cycle and quantity decisions, while the supplier tries to satisfy the retailer's order if the transaction is profitable. For the retailer, factors affecting the profit are mainly manifested in the following two aspects. On the one hand, due to spoilage, freshness of the product reduces; on the other hand, the shortage of the product appears because the quantity of the original ordered product decreases. After the retailer determines the ordering quantity, he submits the order to the supplier, and in return, the supplier orders from the outside.

In a sale process, demand of fresh agricultural products will decrease with the reduction of freshness. As fresh agricultural products are the necessities in the real life, the related demand become more stable, so the common time varying demand function is introduced, namely, $D(t) = \alpha e^{\beta t}$, α is the largest demand rate and $\alpha > 0$, $\beta < 0$ means the demand for fresh agricultural products reduces as the time goes on. Fresh agricultural products are subject to spoilage and man-made losses, but the loss is generally stable. Thus the deterioration rate can be taken as a constant λ, where $0 < \lambda < 1$.

Introduce a deteriorating product inventory model (Dye, 2007)[21]

$$dI(t)/dt = -\alpha e^{\beta t} - \lambda I(t), 0 \le t \le T \quad (1)$$

where $I(t)$ is the level of positive inventory at time t, T is the retailer's ordering cycle, deterioration rate λ of fresh agricultural products is based on the retailer's experience, which comes from the historical data and statistics. It is a very important consideration that reflects the characteristics of this agricultural product. Let $I(T) = 0$, solve the differential equation above, then we can get

$$I(t) = \alpha/(\beta + \lambda)\left(e^{(\beta+\lambda)T - \lambda t} - e^{\beta t}\right), 0 \le t \le T \quad (2)$$

At the beginning of each order cycle, we substitute $t = 0$ into the equation above to get the retailer's order quantity,

$$Q_r = I(0) = \alpha/(\beta+\lambda)\left(e^{(\beta+\lambda)T} - 1\right) \quad (3)$$

The quantity of deteriorating or stale agricultural products in an order cycle is

$$K_r = \int_0^T \lambda I(t)\,dt = \alpha\lambda/(\beta+\lambda)\left[1/\lambda\left(e^{(\beta+\lambda)T} - e^{\beta T}\right) + 1/\beta\left(1 - e^{\beta T}\right)\right] \quad (4)$$

When spoilage is less it is easily overlooked. However, when consumers make a selection during the purchasing process, this selection speeds up the occurrence of deterioration. Usually at the end of a cycle, fresh agricultural products left will be little, so consumers are not willing to buy them at the previous price. Therefore, the retailer generally lowers the price, that is, the salvage value of fresh agricultural products is v.

Develop the retailer's profit function below

$$\overline{z}_r = \left[(1-\ell)Q_r - K_r\right]P_r + K_r v - \ell Q_r s - C_r - H_r - A_r \quad (5)$$

where P_r is the retailer's unit selling price, P_d is the supplier's unit selling price, A_r is the retailer's ordering cost, ℓ is shortage rate, s is unit shortage cost, the purchase cost is $C_r = P_d Q_r (1-\ell) = P_d \alpha\left(e^{(\beta+\lambda)T} - 1\right)(1-\ell)/(\beta+\lambda)$; let h be holding cost per unit time per unit product, then the inventory carrying cost is

$$H_r = h\int_0^T I(t)\,dt = \alpha h/\left[\lambda(\beta+\lambda)\right]e^{(\beta+\lambda)T} - \alpha h/\left[\lambda(\beta+\lambda)\right]e^{\beta T} + \alpha h/\left[\beta(\beta+\lambda)\right]\left(1 - e^{\beta T}\right) \quad (6)$$

The supplier at first buys products from upstream producers, and then sells the products to the downstream retailer, at the same time, the supplier is in charge of the delivery(or commit the delivery cost) from the supplier to the retailer. Therefore, the profit function of the supplier can be expressed as

$$\overline{z}_s = P_d Q_r (1-\ell) - C_s - N_s \tag{7}$$

where the supplier's purchase cost is $C_s = cQ_r(1-\ell)$, c is unit production cost of the supplier, N_s is the supplier's delivery cost.

In the following, we analyze some properties of the retailer from the above functions. For convenience, all proofs in this paper are moved to Appendix.

Proposition 2.1 *The retailer's optimal ordering cycle and the optimal ordering quantity are denoted as follows, respectively*

$T_r^* = \frac{1}{\lambda}\ln\left[(v - P_r - h/\lambda)/(v + \ell P_d - P_d - \ell P_r - \ell s - h/\lambda)\right]$, $Q_r^* = \alpha/(\beta+\lambda)\left(e^{(\beta+\lambda)T_r^*} - 1\right)$.

According to Proposition 2.1, we can directly obtain the following two corollaries.

Corollary 2.1 *The retailer's the optimal ordering cycle and the optimal ordering quantity are decreasing in* λ *and* ℓ *respectively.*

Corollary 2.2 *When* ℓ *increases, the retailer's profit* $\overline{z}_r$ *and the supplier's profit* $\overline{z}_s$ *will decrease.*

Proposition 2.1 and its corollaries show that, when deterioration rate and shortage rate increase, the powerful retailer should consider reducing the order cycle T and/or reducing the ordering quantity so that he can maximize the profit. For the supplier, he has to keep a low shortage rate to maintain his profit.

The total profit function of the supply chain can be expressed as

$$\overline{\Pi} = \overline{z}_r + \overline{z}_s = [(1-\ell)Q_r - K_r]P_r + K_r v + P_d Q_r (1-\ell) - lQ_r s - C_r - C_s - H_r - A_r - N_S \tag{8}$$

The derivatives of the total profit $\overline{\Pi}$ with respect to T are

$\partial\overline{\Pi}/\partial T = \alpha e^{(\beta+\lambda)T}(v + \ell c - c - \ell P_r - \ell s - h/\lambda) - \alpha e^{\beta T}(v - P_r - h/\lambda)$,

$\partial^2\overline{\Pi}/\partial T^2 = \alpha(\beta+\lambda)e^{(\beta+\lambda)T}(v + \ell c - c - \ell P_r - \ell s - h/\lambda) - \alpha\beta e^{\beta T}(v - P_r - h/\lambda)$.

From proof of Proposition 2.1, we obtain $\partial^2\overline{\Pi}/\partial T^2 < 0$, then $\overline{\Pi}$ is concave in T and there exist a maximum point. The optimal order cycle and the optimal ordering quantity are

$\overline{T}' = 1/\lambda \ln\left[(v - P_r - h/\lambda)/(v + \ell c - c - \ell P_r - \ell s - h/\lambda)\right]$, $\overline{Q}' = \alpha/(\beta+\lambda)\left(e^{(\beta+\lambda)T'} - 1\right)$.

Similar to Corollary 2.1, we get that when shortage rate ℓ increases, the supply chain's optimal ordering cycle $\tilde{T}'$ will decrease, and optimal ordering quantity $\tilde{Q}'$ will decrease. Compare T_r^* with $\bar{T}'$, due to $c < P_d, \ell < 1$, so $\ell P_d - P_d < \ell c - c$, that is,

$v + \ell P_d - P_d - \ell P_r - \ell s - h/\lambda < v + \ell c - c - \ell P_r - \ell s - h/\lambda < 0, \qquad v - P_r - h/\lambda < 0$.

Then

$(v - P_r - h/\lambda)/[v + \ell P_d - P_d - \ell P_r - \ell s - h/\lambda] < (v - P_r - h/\lambda)/[v + \ell c - c - \ell P_r - \ell s - h/\lambda]$.

Because of $T > 0$, we derive

$$1 < (v - P_r - h/\lambda)/[v + \ell P_d - P_d - \ell P_r - \ell s - h/\lambda] < (v - P_r - h/\lambda)/[v + \ell c - c - \ell P_r - \ell s - h/\lambda].$$

Thus,

$0 < \ln\{(v - P_r - h/\lambda)/[v + \ell P_d - P_d - \ell P_r - \ell s - h/\lambda]\} < \ln\{(v - P_r - h/\lambda)/[v + \ell c - c - \ell P_r - \ell s - h/\lambda]\}$.

Then $T_r^* < \overline{T}'$. Therefore, we should find some contracts to make the retailer and supplier take more profit.

3. Optimal Ordering Policies under the Revenue Sharing Contract

With the revenue sharing contract, the sequence of the event is described as follows. The supplier charges the retailer at a lower unit price P_w, and in return, the retailer will distribute revenue to the supplier according to a certain proportion $1-\phi\,(0 < \phi \le 1)$. Therefore, the contract parameter can be expressed as $\{P_w, \phi\}$ [22]. At the end of selling period, the product with lower freshness has salvage value v. The product price satisfies $v < P_d < P_r$. Demand for fresh agricultural products is generally stable, so it is assumed that $|\beta| < \lambda < 1$.

Therefore, the retailer's profit can be written as

$$\tilde{z}_r = \phi\{[(1-\ell)Q_r - K_r]P_r + K_r v\} - lQ_r s - C_r' - H_r - A_r \quad (9)$$

where the purchase cost is $C_r' = P_w Q_r(1-\ell)$. The supplier's profit is

$$\tilde{z}_s = (1-\phi)\{[(1-\ell)Q_r - K_r]P_r + K_r v\} + P_w Q_r(1-\ell) - C_s - N_s \quad (10)$$

where the production cost is $C_s = cQ_r(1-\ell)$. Then the total profit of the supply chain is $\tilde{\Pi} = \tilde{z}_r + \tilde{z}_s = \bar{\Pi}$.

From the above expression, we can get the following proposition.

Proposition 3.1 *The supply chain can be coordinated with the revenue-sharing contract satisfying*

$P_w = \left[\phi c(v-P_r) + \phi \ell s(v-P_r)/(1-\ell) - \phi h P_r/\lambda - \ell s(v-P_r)/(1-\ell) + h/\lambda(P_r - c)\right]/(v - P_r - h/\lambda)$ *and* $\phi \in (0,1]$.

Furthermore, P_w *is increasing in* ϕ.

Proposition 3.1 provides the condition that supply chain can be coordinated by the revenue sharing contract when customer demand is stable and time varying, which is a deterministic version and different from the Cachon and Larivere (2005) [22]'s result in which the demand is stochastic. In fact, the wholesale price cannot be too small, since the participants' reservation profits should be nonnegative; therefore, we offer a bound for this case in the following corollary.

Corollary 3.1 *If* $P_w \geq (C_s + N_s)/\left[(1-\ell)Q_r\right]$ *and the both the supplier's and the retailer's reservation profit are zero, we can get the range of* ϕ *is* $(lQ_r s + C_r' + H_r + A_r)/\left[(1-\ell)Q_r P_r - K_r P_r + K_r v\right] < \phi \leq 1$.

Form Corollary 3.1, we derive that the parameter ϕ can help achieve allocation of supply chain profit, specific profit allocation still depends on bargain power of the supplier and the retailer. When they determine their reservation profits (such as zero), they are willing to participate in the revenue sharing contract only when they are both better off.

4. Optimal Ordering Policies under the Quantity Discount Contract

Under the quantity discount contract, the supplier provides a wholesale price according to the retailers' order quantity. The more the retailer orders, the greater discount the supplier offers. Taking the shortage rate into account, the order quantity $Q_r = \alpha\left(e^{(\beta+\lambda)T} - 1\right)/(\beta+\lambda)$ is a function with respect to the order cycle T, so in this quantity discount contract, the supplier's wholesale price P_σ is a function with two variables, order cycle and shortage rate. Establish the retailer's profit function

$$\hat{z}_r = \left[(1-\ell)Q_r - K_r\right]P_r + K_r v - P_\sigma Q_r(1-\ell) - \ell Q_r s - H_r - A_r \quad (11)$$

and the supplier's profit function is

$$\hat{z}_s = P_\sigma Q_r(1-\ell) - cQ_r(1-\ell) - N_s \quad (12)$$

then the total supply chain's profit function is

$$\hat{\Pi} = \hat{z}_r + \hat{z}_s = \left[(1-\ell)Q_r - K_r\right]P_r + K_r v - cQ_r(1-\ell) - \ell Q_r s - H_r - A_r - N_s \quad (13)$$

Proposition 4.1 *Suppose that the supplier and the retailer share the total profit of supply chain in proportion* $\delta/(1-\delta)$, *then we have*

$P_\sigma(Q_r) = (1-\delta)P_r + \delta c + \left[\delta N_s - (1-\delta)(K_r P_r + H_r + A_r - K_r v)\right]/\left[Q_r(1-\ell)\right]$.

Two contracts are both implemented through the payment transfer between the supplier and the retailer. The main difference between them is that, with the revenue sharing contract, the supplier gives a lower wholesale price commitment and the retailer transfers his percentage of revenue in return; while in quantity discount contract with the given profit proportion, the supplier promises a discount to encourage the retailer to order more. The quantitative relationship can be expressed in a corollary.

Corollary 4.1 *When the contract parameter satisfies*

$$\delta = \left[\phi(1-\ell)Q_rP_r - \phi K_rP_r + \phi K_rv - P_wQ_r(1-\ell) - lQ_rs - H_r - A_r\right] / \left[(1-\ell)Q_rP_r - K_rP_r + K_rv - cQ_r(1-\ell) - \ell Q_rs - H_r - A_r - N_s\right]$$

,

both the quantity discount contract and the revenue sharing contract can coordinate the supply chain, participants of supply chain can get the same profit allocation.

From Corollary 4.1, under a certain condition, the revenue sharing contract is equivalent to the quantity discount contract, and equivalency is influenced by shortage and deterioration.

5. Optimal Ordering Policies under the Buy-back Contract

Under the buy-back contract, the supplier charges the retailer at a wholesale price P_ε, but at the end of sale season, the supplier will return the unsold products with unit price b , so the parameter set can be expressed as $\{P_\varepsilon, b\}$. Due to the extension of sale time, fresh agricultural products' value will gradually diminish. Therefore, $b \le P_\varepsilon$ (the return price is lower than previous wholesale price), the retailer cannot get profit from surplus products, which is in line with the actual situation. We can establish the retailer's profit function

$$\breve{z}_r = \left[(1-\ell)Q_r - K_r\right]P_r + K_rb - P_\varepsilon Q_r(1-\ell) - \ell Q_rs - H_r - A_r \tag{14}$$

After buying back the fresh agricultural products, the supplier will reprocess them for sale, since resale price per unit for agricultural products is v , the profit function of the supplier can be expressed as

$$\breve{z}_s = P_\varepsilon Q_r(1-\ell) + K_rv - cQ_r(1-\ell) - K_rb - N_s \tag{15}$$

Then the supply chain's profit is

$$\breve{\Pi} = \breve{z}_r + \breve{z}_s = \left[(1-\ell)Q_r - K_r\right]P_r + K_rv - cQ_r(1-\ell) - \ell Q_rs - H_r - A_r - N_s \tag{16}$$

Therefore we offer the coordinating condition of the supply chain in the following proposition.

Proposition 5.1 *The supply chain can be coordinated when the buy-back contract satisfies*

$$P_\varepsilon = \left[b\left(c + s/(1-\ell) - P_r - s\right) + P_r(v-c) - hc/\lambda - sv\ell/(1-\ell)\right] / \left(v - P_r - h/\lambda\right).$$

Proposition 5.1 shows that when the supply chain is coordinated, shortage and deterioration will influence the coordinating contract. Since $v - P_r - h/\lambda < 0$, when $c + s/(1-\ell) - P_r - s \le 0$, i.e.,$\ell \le (P_r - c)/(P_r - c + s)$, P_ε is an increasing function with respect to buy-back price b , otherwise, P_ε is decreasing function with respect to buy-back price b , that is , if the shortage rate is low, the supplier will raise the wholesale price when increasing the buy-back price, but if the shortage rate is high, the supplier has to reduce the wholesale price to absorb the retailer while increase the buy-back price. When $b > v$, the wholesale price P_ε is increasing in shortage rate ℓ , when $b \le v$, the wholesale price P_ε is decreasing in shortage rate ℓ .

Proposition 5.2 *With parameters satisfying*

$$P_\varepsilon = P_w + \left[(1-\phi)(1-\ell)Q_r - (1-\phi)K_rP_r + K_r(b - \phi v)\right] / \left[Q_r(1-\ell)\right],$$

the optimal ordering quantity under buy-back contract is the same as that under revenue sharing contract, and profit allocation under the two contracts are completely identical.

Proposition 5.2 provides the equivalent condition between the revenue sharing contract and the buy-back contract, notice that this condition is relevant to shortage rate.

6. Numerical Analysis

This section makes numerical analysis to intuitively show effect of deteriorate rate and shortage rate on decision makings, contracts and supply chain performance. For simplicity, some parameters in this section will be set as constants.

6.1. Wholesale Price Contract Case

In the process to assign values, we make some investigations about fresh agricultural products' unit price which refers to some literature related to shortage rate, deterioration rate, cost and demands, so that the simulation result is much in line with actual situations.

Table 1. Parameters Assignment

P_r	P_d	s	v	A_r	c	h	α	β	λ	N_s
4	2.5	1.5	1	20	2	0.1	50	-0.05	0.1	20

The optimal ordering cycle and the related profits range as follows.

Figs. 1-3 are about here

From the figures above, we know that the retailer's and the supplier's profits both decrease as the shortage rate increases. The decline of the retailer's margin profit is more than that of the supplier, which will lead to low efficiency of the supply chain. So, the supply chain should try to reduce the shortage rate to improve the operating performance of the supply chain.

6.2. Impact of shortage and deterioration on ordering policy

Shortage and deterioration are two main factors affecting ordering policies for fresh agriculture products, so we make a further analysis. First, we set deterioration rate $\lambda=0.1$, shortage rate ℓ ranges from 0.02 to 0.5 (the interval is 0.03), then the optimal ordering cycle T', ordering quantity Q' among wholesale price contract and coordinating contracts will change as follows. Note that, as shown in the sections above, the revenue sharing, quantity discount and buy-back contracts are equivalent under certain conditions, then the optimal ordering cycles and quantities under the three contracts are the same respectively. Therefore, we only provide curves of the revenue sharing contracts called coordinating contract.

Figs. 4-6 are about here

From Figure 4 and 5, we get that, when shortage rate ℓ increases, the supply chain's optimal ordering cycle T' and the optimal ordering quantity Q' will both decrease. Compared with the coordinating contract, under the wholesale contract, the optimal order cycle is shorter and the optimal order quantity is larger.

From Figure 6, for the wholesale prices between the supplier and the retailer, except for the quantity discount contract, the wholesale prices under other three contracts are decreasing. That under revenue sharing contract is much lower than those under other contracts since only revenue sharing contract provides very low wholesale price even lower than the supplier's cost. The wholesale price of the quantity discount contract will decrease with the increasing of the order quantity, note that the order quantity is increasing in shortage rate. Therefore, it is reasonable that the wholesale price is increasing in shortage rate as shown in the figure.

Second, we explore the impact of deterioration rate λ, so the shortage rate ℓ should be fixed ($\ell = 0.2$). Then let λ range from 0.1 to 0.8 (the interval is 0.05), then the optimal ordering cycle T' and ordering quantity Q' among different contracts will change as follows.

Figs.7-9 are about here

From Figure7-9, we can derive that, when deterioration rate λ increases, the supply chain's optimal ordering cycle T' and the optimal ordering quantity Q' will both decrease, while the wholesale prices in different contracts will increase. Furthermore, the optimal ordering quantity and cycle in coordinating contracts are higher than those in the wholesale price contract, and the wholesale price in the buy-back contract is comparatively higher than that in others contracts.

Through the above numerical examples, we get some further verifications, the supply chain should reduce the shortage rate and deterioration rate to ensure the whole supply chain profit.

7. Conclusion

In a fresh agricultural product supply chain, both shortage and deterioration are very important elements affecting the decision of the retailer and the supplier. Fresh agricultural products are the necessities of life, shortage of which may cause a rise in price, affecting residents' ordinary life. This article mainly analyzes two factors on fresh agricultural product—shortage and deterioration, and establishes the retailer's profit function on the basis of decentralized decision-making. We get the optimal ordering policies and establish different contract models for the supplier and the retailer. Then we show the impact of shortage and deterioration on optimal decisions and contracts theoretically. Finally, through numerical analysis, we intuitively show different impact of shortage and deterioration on the optimal decisions and wholesale prices with different contracts, it can provide some managerial reference to make order decisions and select contracts for managers.

Premise of this article is that there is only one supplier and one the retailer. In reality, the different retailer will collaborate with multiple suppliers, and factors affecting the decision of retailers and the suppliers are diverse, not just shortage rate and deterioration rate. In addition, there are varieties of supply chain contracts, but in our article, we only discuss several. What's more, in the actual application, there are many ways to research the supply chain coordination. However, only supply chain contracts are studied in our article. Those issues, which are not already involved in this article, would be the further research content for fresh agricultural products supply chain.

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Appendix

Proof of Proposition 2.1. The retailer’s profit function is a function of the order cycle, so we can get the derivative of this function

$$\frac{\partial \overline{z}_r}{\partial T} = \left[(1-\ell)P_r - \ell s - (1-\ell)P_d\right]\alpha e^{(\beta+\lambda)T} + \left(v - P_r\right)\left[\alpha e^{(\beta+\lambda)T} - \alpha e^{\beta T}\right] - \frac{\alpha h}{\lambda} e^{(\beta+\lambda)T} + \frac{\alpha h}{\lambda} e^{\beta T}$$

$$= \alpha e^{(\beta+\lambda)T}(v + \ell P_d - P_d - \ell P_r - \ell s - h/\lambda) - \alpha e^{\beta T}\left(v - P_r - h/\lambda\right).$$

Then, $\frac{\partial^2 \overline{z_r}}{\partial T^2} = \alpha(\beta+\lambda)e^{(\beta+\lambda)T}(v - P_d + \ell P_d - \ell P_r - \ell s - h/\lambda) - \alpha\beta e^{\beta T}(v - P_r - h/\lambda)$.

Because $|\beta| < \lambda < 1$, so $\beta + \lambda > 0, \beta + 1 > 0$ and $\beta < 0, \alpha > 0, h > 0, v < P_d < P_r$, then $-\alpha\beta e^{\beta T}(v - P_r - h/\lambda) < 0$, $v - P_d < 0$, $\ell P_d - \ell P_r < 0$.

Finally, we can get $\frac{\partial^2 \overline{z_r}}{\partial T^2} < 0$, that is, there exists the maximum value for the retailer's profit. Let $\partial \overline{z_r} / \partial T = 0$, we get $T_r^* = \frac{1}{\lambda} \ln \frac{v - P_r - h/\lambda}{v + \ell P_d - P_d - \ell P_r - \ell s - h/\lambda}$. Substitute (9) into (3), we can solve the optimal ordering quantity $Q_r^* = \frac{\alpha}{\beta+\lambda}\left(e^{(\beta+\lambda)T_r^*} - 1\right)$. Then we can get the retailer's profit z_r^*, the supplier's profits z_s^*, as well as the supply chain's profit Π^*.

To simplify the optimal ordering cycle function before the analysis, let $A = v - P_r - h/\lambda < 0$, $B = P_d - P_r - s < 0$, $C = v - P_d - h/\lambda < 0$. Then the optimal order period function on shortage rate is $T(\ell) = \frac{1}{\lambda} \ln \frac{A}{B\ell + C}$. By differentiation, we have $\frac{\partial T(\ell)}{\partial \ell} = -\frac{B}{\lambda(B\ell + C)} < 0$. Thus, $T(\ell)$ is monotone decreasing function on ℓ. When the shortage rate is fixed, there is only one unique optimal ordering cycle that can maximize the retailer's profit. From the property of the exponential function, we know that Q_r is increasing function of the order cycle *T*, it also means that Q_r is monotone decreasing function of the shortage rate ℓ. □

Proof of Proposition 3.1. Derivative of the retailer's profit $\tilde{z}_r$ on order cycle T is

$$\frac{\partial \tilde{z}_r}{\partial T} = \alpha e^{(\beta+\lambda)T}[\phi v - \phi \ell P_r - \ell s - P_w(1-\ell) - h/\lambda] - \alpha e^{\beta T}(\phi v - \phi P_r - h/\lambda)$$

When $\phi v - \phi \ell P_r - \ell s - P_w(1-\ell) - h/\lambda < 0$, we can get $\frac{\partial^2 \tilde{z}_r}{\partial T^2} < 0$, that is, here must be maximum for retailer's profits. Let $\frac{\partial \tilde{z}_r}{\partial T} = 0$, we can evaluate the corresponding order cycle and ordering quantity $T'' = \frac{1}{\lambda} \ln \frac{\phi v - \phi P_r - h/\lambda}{\phi v - \phi \ell P_r - \ell s - P_w(1-\ell) - h/\lambda}$ and $Q'' = \frac{\alpha}{\beta+\lambda}\left(e^{(\beta+\lambda)T''} - 1\right)$. Only when total supply chain profit and the retailer profit achieve maximum value at the same time, can we get the optimal solution. That is, $\frac{1}{\lambda} \ln \frac{v - P_r - h/\lambda}{v + \ell c - c - \ell P_r - \ell s - h/\lambda} = \frac{1}{\lambda} \ln \frac{\phi v - \phi P_r - h/\lambda}{\phi v - \phi \ell P_r - \ell s - P_w(1-\ell) - h/\lambda}$, then the condition to reach coordination for decentralized supply chain under revenue sharing contract is $P_w = \frac{\phi\left[c(v - P_r) + \ell s(v - P_r)/(1-\ell) - hP_r/\lambda\right] - \ell s(v - P_r)/(1-\ell) + (P_r - c)h/\lambda}{v - P_r - h/\lambda}$. Let

$A' = c(v - P_r) + \frac{\ell s(v - P_r)}{1-\ell} - \frac{hP_r}{\lambda}$, $B' = -\frac{\ell s(v - P_r)}{1-\ell} + \frac{h}{\lambda}(P_r - c), C' = v - P_r - \frac{h}{\lambda}$.

Then we can get a simplified formula $P_w = \frac{A'\phi + B'}{C'}$. From above formula we can get that there is a linear relationship between P_w and ϕ. Since $v < P_r$,we have $A' = c(v - P_r) + \frac{\ell s(v - P_r)}{1-\ell} - \frac{hP_r}{\lambda} < 0$, $C' = v - P_r - \frac{h}{\lambda} < 0$. Therefore, P_w is increasing in ϕ.□

Proof of Corollary 3.1. The premise for retailer to participate in the revenue sharing contract is $\tilde{z}_r > 0$. So, we can get $\phi > \frac{lQ_r s + C_r' + H_r + A_r}{[(1-\ell)Q_r - K_r]P_r + K_r v}$. And $(1-\ell)Q_r$ is the actual delivery of fresh agricultural products, K_r is the deterioration , so $(1-\ell)Q_r - K_r > 0$. That is $\phi > \frac{lQ_r s + C_r' + H_r + A_r}{[(1-\ell)Q_r - K_r]P_r + K_r v} > 0$. The same way, supplier's profit should satisfy $\tilde{z}_s > 0$, so we get $\phi < 1 + \frac{(1-\ell)Q_r P_w - C_s - N_s}{[(1-\ell)Q_r - K_r]P_r + K_r v}$. Due to the given condition $P_w \ge \frac{C_s + N_s}{(1-\ell)Q_r}$, *i.e.*, $(1-\ell)Q_r P_w - C_s - N_s > 0$, then $1 + \frac{(1-\ell)Q_r P_w - C_s - N_s}{[(1-\ell)Q_r - K_r]P_r + K_r v} > 1$. Consider that the original range of ϕ is $(0,1]$, so we can get $\frac{lQ_r s + C_r' + H_r + A_r}{[(1-\ell)Q_r - K_r]P_r + K_r v} < \phi \le 1$. □

Proof of Proposition 4.1. Assume that the retailer's profit is a percentage δ of total profit in supply chain, the retailer's profit function also can be expressed as

$$\hat{z}_r = \delta[(1-\ell)Q_r P_r - K_r P_r + K_r v - cQ_r(1-\ell) - \ell Q_r s - H_r - A_r - N_s].$$

Comparing to the original expression $\hat{z}_s = P_\sigma Q_r(1-\ell) - cQ_r(1-\ell) - N_s$, we can get the solution

$$P_\sigma(Q_r) = (1-\delta)P_r + \delta c + \frac{\delta N_s - (1-\delta)(K_r P_r + H_r + A_r - K_r v)}{Q_r(1-\ell)}$$

. □

Proof of Corollary 4.1. With the revenue sharing contract, the retailer's profit function is

$$\tilde{z}_r = \phi[(1-\ell)Q_r P_r - K_r P_r + K_r v] - P_w Q_r(1-\ell) - lQ_r s - H_r - A_r,$$

and for the quantity discount contract, the retailer's profit function is

$$\hat{z}_r = \delta[(1-\ell)Q_r P_r - K_r P_r + K_r v - cQ_r(1-\ell) - \ell Q_r s - H_r - A_r - N_s].$$

Then, we know that if $\delta = \frac{\phi[(1-\ell)Q_r P_r - K_r P_r + K_r v] - P_w Q_r(1-\ell) - lQ_r s - H_r - A_r}{(1-\ell)Q_r P_r - K_r P_r + K_r v - cQ_r(1-\ell) - \ell Q_r s - H_r - A_r - N_s}$ .Then $\tilde{z}_r = \hat{z}_r$.

Therefore, we can get that the optimal ordering quantity under the quantity discount contract is exactly the same as that under the revenue sharing contract, and the coordination effects for supply chain profit are identical.

Proof of Proposition 5.1. Derivative of the supply chain's profit $\breve{\Pi}$ with respect to the order cycle T is $\frac{d\breve{\Pi}}{dT} = \alpha e^{(\beta+\lambda)T}(\eta + \ell c - c - \ell P_r - \ell s - h/\lambda) - \alpha e^{\beta T}(\eta - P_r - h/\lambda)$.

Then $\frac{d^2\breve{\Pi}}{dT^2} = \alpha(\beta+\lambda)e^{(\beta+\lambda)T}(\eta + \ell c - c - \ell P_r - \ell s - h/\lambda) - \alpha\beta e^{\beta T}(\eta - P_r - h/\lambda)$.

Because $\frac{d^2\tilde{\Pi}}{dT^2} < 0$, we know that there must be maximum profit for the total supply chain. Let $\frac{d\breve{\Pi}}{dT} = 0$, then we can evaluate the corresponding order cycle and ordering quantity

$$\breve{T}' = \frac{1}{\lambda}\ln\frac{\eta - P_r - h/\lambda}{\eta + \ell c - c - \ell P_r - \ell s - h/\lambda}, \quad \breve{Q}' = \frac{\alpha}{\beta+\lambda}\left(e^{(\beta+\lambda)\breve{T}'} - 1\right).$$

Derivative of the retailer's profit $\breve{z}_r$ is

$$\frac{d\breve{z}_r}{dT} = \alpha e^{(\beta+\lambda)T}[b - \ell P_r - \ell s - P_\varepsilon(1-\ell) - h/\lambda] - \alpha e^{\beta T}(b - P_r - h/\lambda).$$

Due to $\frac{d^2\breve{z}_r}{dT^2} < 0$, There must be maximum for the retailer's profit. Let $\frac{d\breve{z}_r}{dT} = 0$, we can get the corresponding order cycle and ordering quantity

$$\breve{T}'' = \frac{1}{\lambda}\ln\frac{b - P_r - h/\lambda}{b - \ell P_r - \ell s - P_\varepsilon(1-\ell) - h/\lambda}, \quad \breve{Q}'' = \frac{\alpha}{\beta+\lambda}\left(e^{(\beta+\lambda)\breve{T}''} - 1\right).$$

Let $\breve{T}' = \breve{T}''$, we can get the relationship between wholesale price P_ε and buy-back price b as follows.

$$P_\varepsilon = \frac{b(c + s/(1-\ell) - P_r - s) + P_r(\eta - c) - hc/\lambda + s\eta(1 - 1/(1-\ell))}{\eta - P_r - h/\lambda}.$$

Proof of Proposition 5.2. Under the buy-back contract, the supply chain's profit is

$$\breve{\Pi} = \breve{z}_r + \breve{z}_s = \left[(1-\ell)Q_r - K_r\right]P_r + K_r\eta - cQ_r(1-\ell) - \ell Q_r s - H_r - A_r - N_s.$$

Under the revenue sharing contract, the supply chain's profit is

$$\tilde{\Pi} = \tilde{z}_r + \tilde{z}_s = [(1-\ell)Q_r - K_r]P_r + K_r v - cQ_r(1-\ell) - lQ_r s - H_r - A_r - N_S.$$

On the premise of $\eta = v$,we have $\breve{\Pi} = \tilde{\Pi}$. From the retailer's profit functions with two contracts, when $P_\varepsilon = P_w + \frac{(1-\phi)\left[(1-\ell)Q_r - K_r\right]P_r + K_r(b - \phi v)}{Q_r(1-\ell)}$,

we get $\breve{z}_r = \tilde{z}_r$, coordination effects of the buy-back contract will be identical as these of the revenue sharing contract.

All figures are listed as follows.

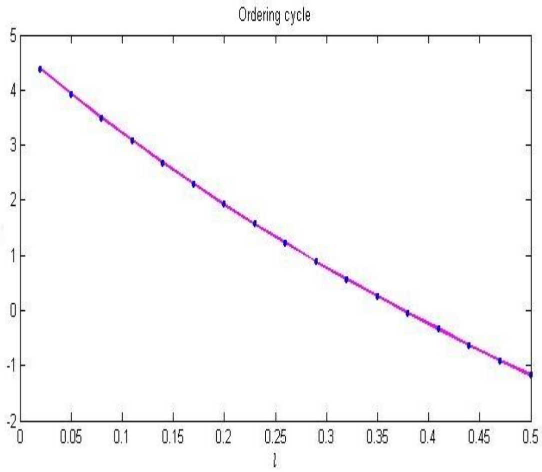

Figure 1. The Optimal Ordering Cycle Versus l

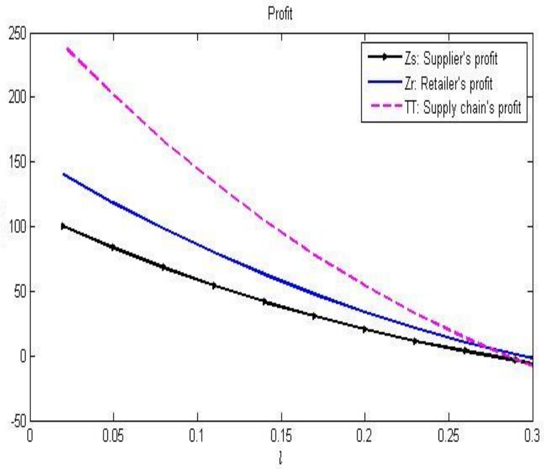

Figure 2. Profits of Supplier, Retailer and Supply Chain Versus l

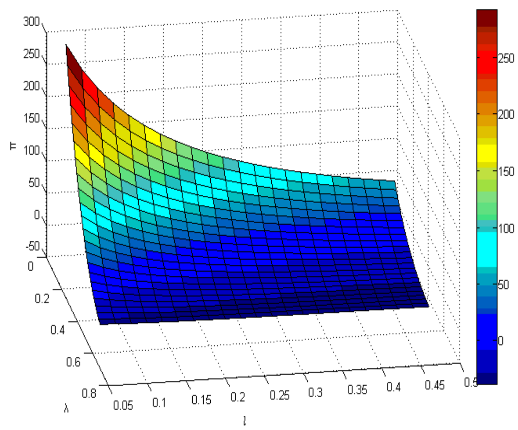

Figure 3. Supply Chain Profit Versus l and λ

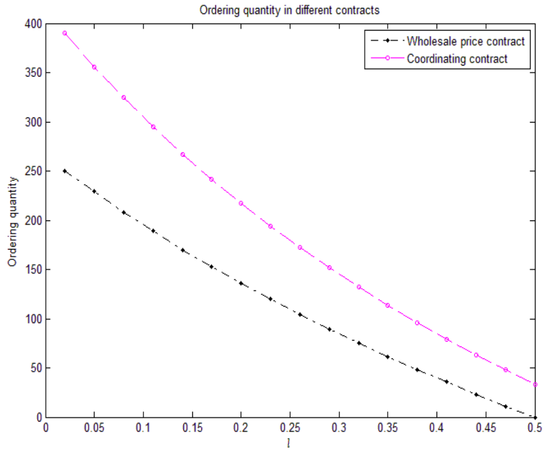

Figure 4. The Optimal Ordering Quantity Versus l

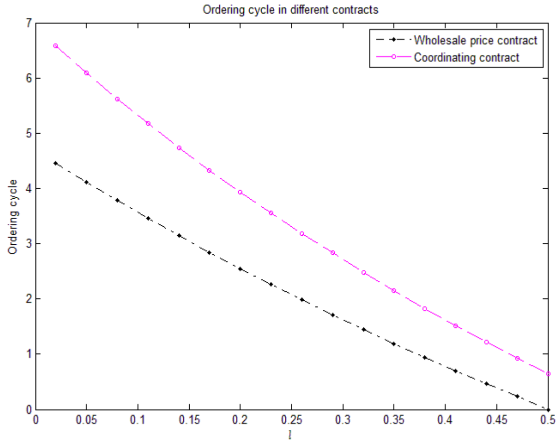

Figure 5. The Optimal Ordering Cycle Versus l

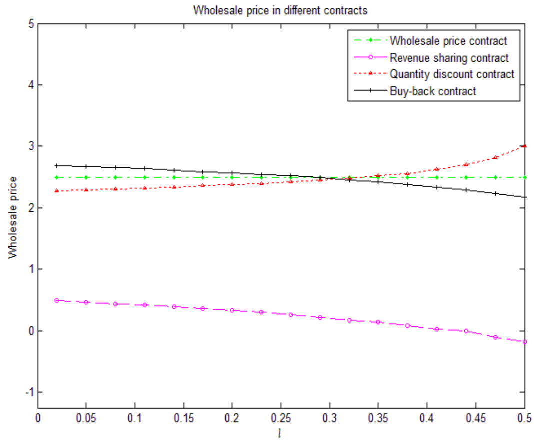

Figure 6. The Wholesale Price in Different Contracts Versus l

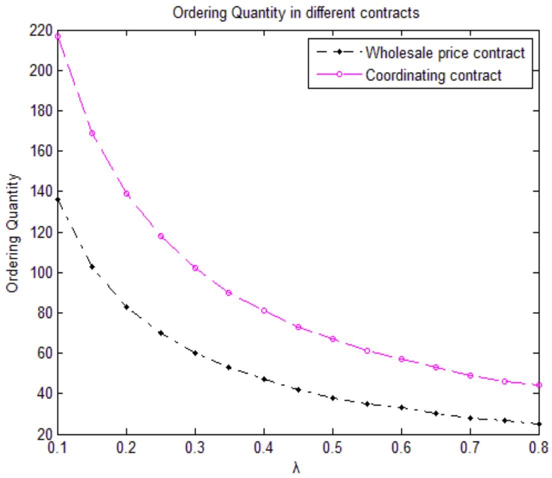

Figure 7. Optimal Ordering Quantity Versus λ

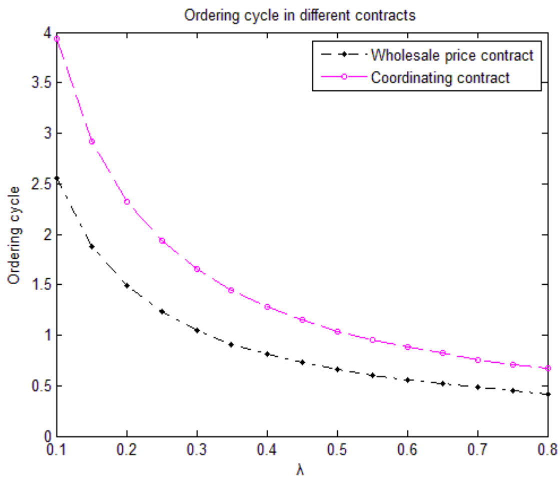

Figure 8. Optimal Ordering Cycle Versus λ

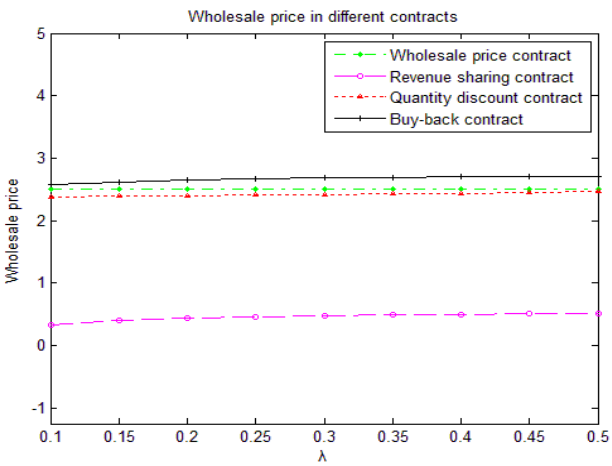

Figure 9. Wholesale Price Versus λ

