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Optimal Linear Filter for Multi-rate Singular System

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Abstract. This paper is concerned with the optimal linear filtering problem for singular system with multiple rates. The state is updated at the higher rate and the measurement is sampled at the lower rate. The original singular system is transformed into fast and slow two subsystems. By introducing the "dummy" random variable, the filter for the slow subsystem is derived. Further, the filter for the original system is obtained based on the filters of the two subsystems.

Keywords: Multi-rate; Singular system; Filter.

1 Introduction

In recent years, the filtering problem for systems with multiple rates has attracted lots of attention due to the impossibility to sample all the physical signals at one single rate in many complicated practical systems. For multi-rate systems, the first important study goes back to the switch decomposition technique proposed by Kranc [1]. So far, many useful filtering strategies are proposed [2-6]. But, the computational cost of the above filtering strategies is high since they are given by state/measurement augmentation. However, the above results are given for the normal multi-rate system. In this article, we consider the filtering problem for multi-rate singular system. the multi-rate filtering problem is transformed into an equivalent single rate filtering problem. Then, the linear filter for the singular system is proposed. The proposed filter can reduce the computational cost since the state augmentation is avoided.

2 Problem Formation

Consider the following linear stochastic singular multi-rate system

$$Ax(tb+b) = Bx(tb) + Cw(tb) \tag{1}$$

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$$z(kb_0) = Dx(kb_0) + \underline{v}(kb_0), \quad b_0 = cb \tag{2}$$

where $x(tb) \in \mathbb{R}^n$ is the state vector at tb time instant, $z(kb_0) \in \mathbb{R}^m$ is the measured output at kb_0 time instant, A is a singular square matrix, i.e. $\mathrm{rank}(A) = n_0 < n$, B, C and D are the constant matrices with suitable dimensions. $w(tb) \in \mathbb{R}^q$ and $\underline{v}(kb_0) \in \mathbb{R}^m$ are correlated white noises with variance matrices R^w, $\underline{R}^v$ and cross-covariance matrices $\mathrm{E}[w(kb_0)\underline{v}^{\mathrm{T}}(kb_0)] = \underline{S}$, respectively. The state $x(tb)$ is updated at a higher rate with a period b and the measurement $z(kb_0)$ is sampled at a lower rate with a period $b_0 = cb$ where c is a positive integer. The initial state vector $x(0)$ is uncorrelated with $w(tb)$ and $\underline{v}(kb_0)$, and satisfies $\mathrm{E}\{x(0)\} = \sigma$ and $\mathrm{E}\{(x(0)-\sigma)(x(0)-\sigma)^{\mathrm{T}}\} = \Omega$.

From [7], there exist nonsingular matrices M and N, such that

$$MAN = \begin{bmatrix} A^{(1)} & 0 \\ A^{(2)} & 0 \end{bmatrix}, \quad MBN = \begin{bmatrix} B^{(1)} & 0 \\ B^{(2)} & B^{(3)} \end{bmatrix}, MC = \begin{bmatrix} C^{(1)} \\ C^{(2)} \end{bmatrix}, \quad DN = \begin{bmatrix} D^{(1)} & D^{(2)} \end{bmatrix}$$

where $A^{(1)}$ is a nonsingular lower-triangular matrix with the dimension $n^{(1)} \times n^{(1)}$, $B^{(1)}$ is a quasi lower-triangular matrix with the dimension $n^{(1)} \times n^{(1)}$, $B^{(3)}$ is a nonsingular lower-triangular matrix with the dimension $n^{(2)} \times n^{(2)}$. By introducing the transformation $x(tb) = N[x^{(1)\mathrm{T}}(tb), x^{(2)\mathrm{T}}(tb)]^{\mathrm{T}}$, where $x^{(1)}(tb) \in R^{n^{(1)}}$, $x^{(2)}(tb) \in R^{n^{(2)}}$, then systems (1) and (2) can be transformed into the following systems

$$\begin{cases} x^{(1)}(tb+b) = \bar{A}x^{(1)}(tb) + \bar{B}w(tb) \\ z(kb_0) = \bar{D}x^{(1)}(kb_0) + \bar{v}(kb_0) \end{cases} \tag{3}$$

$$x^{(2)}(tb) = Ux^{(1)}(tb) + Rw(tb) \tag{4}$$

where $\bar{A} = (A^{(1)})^{-1}B^{(1)}$, $\bar{B} = (A^{(1)})^{-1}C^{(1)}$, $\bar{D} = D^{(1)} + D^{(2)}U$, $U = (B^{(3)})^{-1}A^{(2)}(A^{(1)})^{-1}B^{(1)} - (B^{(3)})^{-1}B^{(2)}$, $\bar{v}(kb_0) = Tw(kb_0) + \underline{v}(kb_0)$, $T = D^{(2)}R$, $R = (B^{(3)})^{-1}A^{(2)}(A^{(1)})^{-1}C^{(1)} - (B^{(3)})^{-1}C^{(2)}$.

Also, we have the following statistical property

$$\mathrm{E}[w(kb_0)\bar{v}^{\mathrm{T}}(kb_0)] = R^wT^{\mathrm{T}} + \underline{S} = \bar{S},$$

$$\mathrm{E}[\bar{v}(kb_0)\bar{v}^{\mathrm{T}}(kb_0)] = TR^wT^{\mathrm{T}} + T\underline{S} + \underline{S}^{\mathrm{T}}T^{\mathrm{T}} + \underline{R}^v = \bar{R}^v$$

Now, original singular multi-rate system (1)-(2) is transformed into the normal system with multiple sampling rates.

3 Optimal Linear Filter

In this section, we first transform the multi-rate fusion estimation problem into a single rate fusion estimation problem. Similar [6], we introduce the following white Bernoulli distributed variable $\xi(tb)$ with

$$\xi(tb) = \begin{cases} 1, & tb = kb_0 \\ 0, & \text{else} \end{cases}$$

Based on $\xi(tb)$, we have

$$y(tb) = \begin{cases} z(kb_0), & \xi(tb) = 1 \\ 0, & \text{else} \end{cases}, \quad v(tb) = \begin{cases} \bar{v}(kb_0), & \xi(tb) = 1 \\ 0, & \text{else} \end{cases},$$

$$F(tb) = \begin{cases} \bar{D}, & \xi(tb) = 1 \\ 0, & \text{else} \end{cases}, \quad t, k = 1, 2, \cdots$$

From the above definition, we see that $\xi(tb) = 1$ denotes $y(tb) = z(tb)$ and $\xi(tb) = 0$ denotes $y(tb) = 0$. Then system (3) can be transformed into the following single rate system:

$$x^{(1)}(tb + b) = \bar{A}x^{(1)}(tb) + \bar{B}w(tb) \tag{5}$$

$$y(tb) = F(tb)x^{(1)}(tb) + v(tb) \tag{6}$$

Observe that systems (5) and (6) are transformed into the single rate system. Therefore, the filter $\hat{x}^{(1)}(tb \mid tb)$ and the corresponding estimation error variance matrix $P^{(1)}(tb \mid tb - b)$ for the sate $x^{(1)}(tb)$ can be obtained using the results of [6]. In the following section, we shall derive the filter $\hat{x}^{(2)}(tb \mid tb)$ for the sate $x^{(2)}(tb)$ based on the random variables $(\xi(tb), \xi(tb - b), \cdots, \xi(0))$ and $\hat{x}^{(1)}(tb \mid tb)$.

Theorem 1. The filter for system (4) is computed by

$$\hat{x}^{(2)}(tb \mid tb) = U\hat{x}^{(1)}(tb \mid tb) + R\hat{w}(tb \mid tb) \tag{7}$$

$$\hat{w}(tb \mid tb) = \xi(tb)\bar{S}Q_{\varepsilon}^{-1}(tb)\varepsilon(tb) \tag{8}$$

$$P^{(2)}(tb \mid tb) = G(tb)P^{(1)}(tb \mid tb - b)G^{\mathrm{T}}(tb) + H(tb)\begin{bmatrix} R^{w} & \bar{S} \\ \bar{S}^{\mathrm{T}} & \bar{R}^{v} \end{bmatrix}H^{\mathrm{T}}(tb) \tag{9}$$

where $G(tb) = U - \xi(tb)(UK(tb) + R\bar{S}Q_{\varepsilon}^{-1}(tb))F(tb)$, $H(tb) = \left[R, \quad -\xi(tb)(UK(tb) + R\bar{S}Q_{\varepsilon}^{-1}(tb))\right]$. $\hat{w}(tb \mid tb)$ is the white noise filter, $P^{(2)}(tb \mid tb)$ is the filtering error variance matrix for $x^{(2)}(tb)$.

Proof: Taking projection of both sides of (4) onto the linear space $L(y(tb), \cdots, y(b))$, we have (7). The white noise filter $\hat{w}(tb \mid tb)$ is obtained by applying the projection theory [8]

$$\hat{w}(tb \mid tb) = \hat{w}(tb \mid tb - b) + \mathrm{E}[w(tb)\varepsilon^{\mathrm{T}}(tb)]Q_{\varepsilon}^{-1}(tb)\varepsilon(tb) \tag{10}$$

where the white noise one-step predictor $\hat{w}(tb \mid tb - b)$ is zero vector. From the results of [6], we have the following innovation sequences

$$\varepsilon(tb) = \xi(tb)v(tb) + F(tb)\tilde{x}^{(1)}(tb \mid tb - b) \tag{11}$$

From $w(tb) \perp x^{(1)}(tb)$, $w(tb) \perp \hat{x}^{(1)}(tb \mid tb - b)$ and $(\xi(tb))^2 = \xi(tb)$, we have

$$\mathrm{E}[w(tb)\varepsilon^{\mathrm{T}}(tb)] = \xi(tb)\mathrm{E}[w(tb)v^{\mathrm{T}}(tb)] = \xi(tb)\bar{S} \tag{12}$$

Substituting (12) into (10), the (8) is obtained.

From (4) and (7), we have the filtering error equation for state $x^{(2)}(tb)$

$$\tilde{x}^{(2)}(tb \mid tb) = U(x^{(1)}(tb) - \hat{x}^{(1)}(tb \mid tb)) + R(w(tb) - \hat{w}(tb \mid tb)) \quad (13)$$

From [6], and using (8), (10) and $(\xi(tb))^2 = \xi(tb)$, we can rewrite (13)as

$$\tilde{x}^{(2)}(tb \mid tb) = U\tilde{x}^{(1)}(tb \mid tb-b) + Rw(tb) - \xi(tb)(UK(tb) + R\bar{S}Q_{\varepsilon}^{-1}(tb))\varepsilon(tb) \quad (14)$$

Substituting (11) into (14), the filtering error can be further rewritten as

$$\tilde{x}^{(2)}(tb \mid tb) = G(tb)\tilde{x}^{(1)}(tb \mid tb-b) + H(tb)[w^{\mathrm{T}}(tb) \quad v^{\mathrm{T}}(tb)]^{\mathrm{T}} \quad (15)$$

where $G(tb)$ and $H(tb)$ are defined as above. Substituting (15) into $P^{(2)}(tb \mid tb) = \mathrm{E}[\tilde{x}^{(2)}(tb \mid tb)\tilde{x}^{(2)\mathrm{T}}(tb \mid tb)]$, we have (9).

Remark 1: From the definition $x(tb) = N[x^{(1)\mathrm{T}}(tb), x^{(2)\mathrm{T}}(tb)]^{\mathrm{T}}$, we have the filter $\hat{x}(tb \mid tb) = N[\hat{x}^{(1)\mathrm{T}}(tb \mid tb), \hat{x}^{(2)\mathrm{T}}(tb \mid tb)]^{\mathrm{T}}$ for the original systems (1) and (2).

4 Conclusion

In this paper, a multi-rate filtering problem for linear stochastic singular system is studied. The considered system includes two rates: the sate updating rate and the measurement sampling rate, where the measurement sampling rate is a multiple of state updating rate. We derive the least square filter at the state updating points by introducing "dummy" random variable.

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