

## Accumulated Error Correction of Strip Stereo Models Connection without GCPs

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### Abstract

*The commonly used accumulated error correction of strip stereo models connection need a number of GCPs (Ground Control Points), which is unsuitable for low-altitude photogrammetry with a large number of images. In this paper, a novel approach of accumulated error correction of strip stereo models connection without GCPs was proposed. The accumulated error is adjusted by changing reference image and nonlinear polynomial method. Firstly the middle image of a strip is used as reference image. Then the accumulated error of baseline parameters are adjusted by inverse distance weighted method. Finally the rotation angles of relative orientation are adjusted by polynomial formula between the fitted angles and ideal statistics. Experimental results show that the approach is effective and practical, and can significantly reduce the distortion and accumulated error of model connection, may be suitable to implement strip aerial triangulation in low-altitude photogrammetry.*

**Keywords:** *stereo model, accumulated error, reference image, ground control point (GCP), strip aerial triangulation*

### 1. Introduction

In aerial photogrammetry field, strip aerial triangulation is the most widely used method, especially be employed frequently in low-altitude photogrammetry in recent years, initial values are provided by strip aerial triangulation for block adjustment. In the strip aerial triangulation constructed process, the sequential stereopairs relative orientation and scale transfer to ensure uniform scale along the strip [1], the first stereo model of relative orientation is referenced in every strip, then stereo models connection is implemented by pass points and strip ties, which are transferred along the line.

In strip stereo models connected process, the accumulated error is likely to be caused by systematic and random errors propagation [2], and may get worse with later stereo model. The accumulated error will generate side-effects such as strip distortion, differential scale etc. Unfortunately, the relationship of strip distortion cannot be exactly formulated by mathematical formula. The strip stereo models connection may even fail for accumulated error, which will be tremendously magnified by increased numbers of strip images. The commonly used accumulated error correction of strip stereo models is implemented by nonlinear polynomial, which is established by minimizing the square sum of residual error of GCPs and measured strip coordinates with least square algorithm. A number of polynomial interpolation formulations were developed for adjusting the accumulated error [1, 3].

Compared with traditional aerial photogrammetry, low-altitude photogrammetry has great differences in platform, flight altitude, sensor, pose *etc.*, [4-5], low-altitude photogrammetry has many advantages: high resolution, high efficiency, high flexibility and low cost, and become a hot topic throughout the world and a popular field of

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research. A single image may only cover a small area, so it demands hundreds of images to have the whole interesting area be covered completely. And image has irregular overlap, big rotation angle etc. Therefore, the propagating path of accumulated error may be longer in low-altitude photogrammetry field, there are not enough GCPs to establish error equation for diminishing accumulated error of strip stereo models connection in aerial triangulation, in which the commonly used method may be unsuitable in low-altitude photogrammetry. Among the most popular aerial triangulation and the accumulated error correction are supported by GPS/INS [6-8].

To minimize dependence on GCPs and GPS/INS, a novel approach of accumulated error correction of strip stereo models connection without GCPs was proposed. The accumulated error is adjusted by changing reference image and nonlinear polynomial interpolation formulation. Firstly the middle image of a strip is used as reference image. Then the accumulated error of baseline parameters are adjusted by inverse distance weighted method. Finally the rotation angles of relative orientation are adjusted by polynomial formula between the fitted angles and ideal statistics.

This paper is organized as follows. In Section 2, strip stereo models connection and accumulated error are introduced. Section 3 describes accumulated error correction of strip stereo models connection without GCPs. Section 4 shows the experimental results in low-altitude photogrammetric process. Finally the conclusions are given in Section 5.

## 2. Strip Stereo Models Connection and Accumulated Error

### 2.1. Strip Stereo Models Connection

In aerial photogrammetric process, the first image of spatial auxiliary coordinate system is considered as reference frame in strip stereo models connection, and every stereo model is adjusted to parallel with reference frame by relative orientation. However, the origin and scale of coordinates in stereo models may be different. Every stereo model must be also transferred to ensure a uniform scale and coordinate system along the strip. On the basis of equal elevation in overlapping range of stereopair, starting from first stereo model, the coordinate origin and scale of every stereo model are transferred to the previous model one by one, thus the model fits into a uniform reference frame.

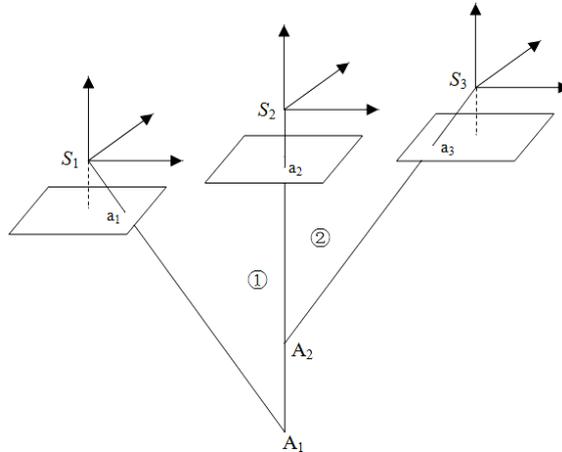
The stereo models connection is performed in different scale (as shown in Figure 1). The scale of model ① and ② are not the same, object spatial point A is forward intersected to A1 and A2 in model ① and ② respectively. It is necessary to normalize model ② to ① for coinciding A1 with A2. The scale  $k$  for transferring model ② to ① is given as follows:

$$k = \frac{S_2 A_1}{S_2 A_2} \quad (1)$$

To achieve a uniform scale, the spatial auxiliary coordinate and baseline  $B(b_u, b_v, b_w)$  of every stereo model are multiplied by  $k$ , so every three-dimensional point in  $S_2$  can be represented in  $S_1$  as follows:

$$\begin{cases} (U)_{S_{2,1}} = (U)_{S_1} + kMN_1 u_1 \\ (V)_{S_{2,1}} = (V)_{S_1} + \frac{1}{2}(kMN_1 v_1 + kMN_2 v_2 + kMb_v) \\ (W)_{S_{2,1}} = (W)_{S_1} + kMN_1 w_1 \end{cases} \quad (2)$$

where  $(U)_{S_{2,1}}, (V)_{S_{2,1}}, (W)_{S_{2,1}}$  is the three-dimensional point in  $S_2$  normalized to  $S_1$ ,  $(U)_{S_1}, (V)_{S_1}, (W)_{S_1}$  is the coordinate in previous stereo model  $S_1$ ,  $u_1, v_1, w_1$  and  $u_2, v_2, w_2$  is the spatial auxiliary coordinate of  $S_1$  and  $S_2$  respectively,  $N_1, N_2$  are the scaling factor of projecting ray in left and right images,  $M$  is the scaling denominator of first stereo model.



**Figure 1. Difference Scale Stereo Model Connection**

## 2.2. Accumulated Error

Ideally, if no error exists, the commonly used strip stereo models connection is available to implement strip aerial triangulation. Unfortunately, there are systematic and random errors in strip stereo models connected process, which can cause accumulated error, from which side-effects such as strip distortion, differential scale *etc.*, will be generated. The main cause of accumulated error has the following aspects:

- (a). the lateral and azimuth deviation in strip orientation may be caused by the oriented error from the deviation of component  $db_v$  and  $d\kappa$  in baseline  $B$  and yaw angle respectively.
- (b). the elevation changes and strip distortion may be caused by the oriented error from the deviation of component  $db_w$  and  $d\varphi$  in baseline  $B$  and pitch angle respectively.
- (c). the closed error of spatial point coordinate or transmitted error of scale may be caused by the deviation of component  $db_u$  in baseline  $B$ .
- (d). the elevation error at the edge of strip model may be caused by the deviation of component  $d\omega$  in roll angle.

However, the relationship of strip distortion cannot be exactly formulated by mathematical formula. Generally, strip model is considered as a adjusted unit, and nonlinear formula is implemented for establishing the adjustment model in the commonly used method of strip models connection, which estimates strip distortion using GCPs and computes nonlinear correction parameters, then strip distortion can be corrected by nonlinear polynomial functional model using data fitting method, in which the barycentric coordinates  $\bar{X}, \bar{Y}, \bar{Z}$  of pass points are considered as observations. In the case of  $X$ , the barycentric coordinates of GCP is  $\bar{X}_{GCP}, \bar{Y}_{GCP}, \bar{Z}_{GCP}$ , the equation of accumulated error correction can be represented as:

$$V = BX - L \quad (3)$$

$$\text{where } V = [-v_{x_1}, -v_{x_2}, \dots, -v_{x_n}]^T, B = \begin{bmatrix} 1 & \bar{X}_1 & \bar{Y}_1 & \bar{X}_1^2 & \bar{X}_1\bar{Y}_1 & \bar{X}_1^3 & \bar{X}_1^2\bar{Y}_1 \\ 1 & \bar{X}_2 & \bar{Y}_2 & \bar{X}_2^2 & \bar{X}_2\bar{Y}_2 & \bar{X}_2^3 & \bar{X}_2^2\bar{Y}_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & \bar{X}_n & \bar{Y}_n & \bar{X}_n^2 & \bar{X}_n\bar{Y}_n & \bar{X}_n^3 & \bar{X}_n^2\bar{Y}_n \end{bmatrix},$$

$$X = [a_0, a_1, a_2, a_3, a_4, a_5, a_6]^T, L = [l_{x_1}, l_{x_2}, \dots, l_{x_n}]^T, l_x = \bar{X}_{GCP} - \bar{X}.$$

As can be seen above, the commonly used accumulated error correction needs many GCPs to establish error equation. Especially, in low-altitude photogrammetry, a surveyed area may cover hundreds of images and image has irregular overlap, big rotation angle *etc.*, which can tremendously magnify the accumulated error. Therefore, it needs a large number of GCPs to diminish accumulated error in low-altitude photogrammetry.

### 3. Accumulated Error Correction without GCPs

In the previous section, we know that the commonly used accumulated error correction is unsuitable for low-altitude photogrammetry. In this section, a novel approach of accumulated error correction of strip stereo models connection without GCPs is presented. This paper mainly includes two aspects for adjusting accumulated error:

- (a). the middle image of a strip is considered as reference for implementing strip aerial triangulation, then accumulated error can be propagated to two sides rather than one side, the strip distortion can be also adjusted from middle image to two sides.
- (b). the photogrammetric routes are planned by equal interval on the basis of overlap between stereo images in traditional vertical photogrammetry. On the basis of that every baseline  $B(b_u, b_v, b_w)$  of a strip can link to a line, starting from the cumulative

statistics  $\left( \left| \sum_{i=1}^n b_{u_i} \right|, \left| \sum_{i=1}^n b_{v_i} \right|, \left| \sum_{i=1}^n b_{w_i} \right| \right)$  of baseline  $B(b_u, b_v, b_w)$ , finding spatial

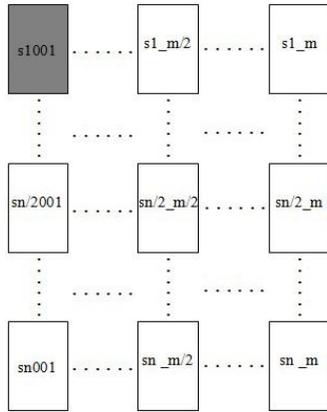
distribution of baseline  $B$ , whose component  $b_u$  is determined as  $b_{u_0}$ . In the ideal case, the cumulative value of  $b_u$  in a strip is  $nb_{u_0}$ , in which the error of actual and ideal value can be adjusted to every stereopair. Meanwhile, ideally the three angles of pitch  $\varphi$ , roll  $\omega$ , yaw  $\kappa$  is 0 ( $\varphi = \omega = \kappa = 0$ ), so the equation of fitted angles by quadratic polynomial can be represented as:

$$\begin{aligned} f_{|\varphi|} &= a_0 + a_1x + a_2x^2 \\ f_{|\omega|} &= b_0 + b_1x + b_2x^2 \\ f_{|\kappa|} &= c_0 + c_1x + c_2x^2 \end{aligned} \tag{4}$$

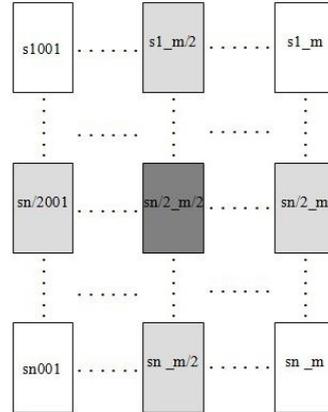
where  $(a_0, a_1, a_2), (b_0, b_1, b_2), (c_0, c_1, c_2)$  is the fitted coefficients of pitch  $\varphi$ , roll  $\omega$ , yaw  $\kappa$  respectively. In order to avoid the offsetting effect caused by positive and negative values in accumulated error, the absolute values of angle are employed to fit functional curve. Then the fitted quadratic polynomials are mapped to the ideal function of angles, and angles can be corrected by the computed parameters on the basis of mapping relation.

After completing strip relative orientation, the middle image of strip is considered as reference. Strip aerial triangulation network can be constructed by strip ties and

image spatial auxiliary coordinate transformation, then all images are transferred to the uniform coordinate in surveyed area. The first image of strips is considered as reference for block adjustment (as shown in Figure 2) in the commonly used method, while the middle image is considered as reference in the proposed method (as shown in Figure 3).



**Figure 2. Aerial Triangulation Network with First Image for Reference**



**Figure 3. Aerial Triangulation Network with Middle Image for Reference**

\*note: s is strip, m is the number of images in a strip, and n is the number of strips

#### 4. Experiments

In this section, the experiments focus on accumulated error correction of strip stereo models connection without GCPs, employing low-altitude images sized  $3744 \times 5616$  from Cannon EOS 5D Mark II camera. Moreover, the proposed accumulated error corrected approach is compared to the commonly used method, whose experimental results are shown in Figure 4 (images of two strips). As can be seen, the accumulated error and strip distortion caused by the commonly used method are propagated from left to right along the direction of increasing stereo images.



Strip 1



strip 2

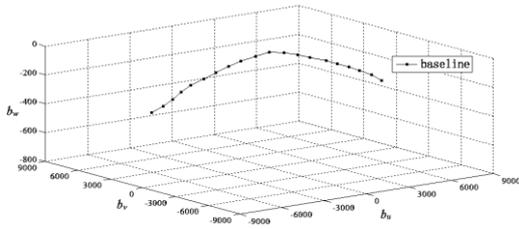
**Figure 4. The Single Strip Distortion Caused by the Accumulated Error with First Image Considered as Reference using the Commonly Used Method**

Strip 2 of Figure 4 is employed for further testing, the middle image is considered as reference, the experimental result is shown in Figure 5. Compared with Figure 4, the accumulated error and strip distortion are propagated to two sides. The spatial distribution of baseline  $B$  in the single strip is shown in Figure 6. The fitted curves of pitch  $\varphi$ , roll  $\omega$ , yaw  $\kappa$  using quadratic polynomial are shown in Figure 7, 8, 9. The transformed parameters can be computed by the relation between the fitted quadratic polynomials and ideal function. And the accumulated error of angles can be adjusted on the basis of the transformed parameters.

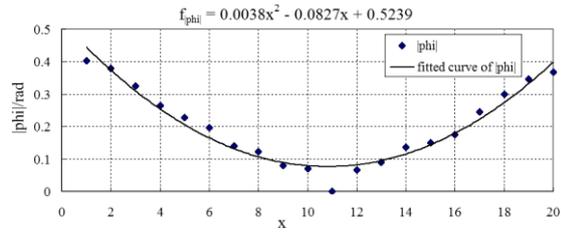
The accumulated error can be adjusted via two aspects of the proposed approach, the experimental result of images stitching is shown in Figure 10.



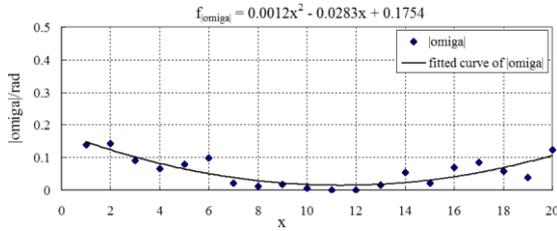
**Figure 5. The Single Strip Distortion Caused by the Accumulated Error with Middle Image Considered as Reference using the Commonly Used Method**



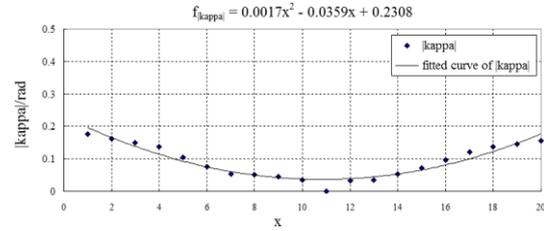
**Figure 6. The Spatial Distribution of Baseline in the Single Strip**



**Figure 7. The Fitted Curve of Absolute Pitch Angle**



**Figure 8. The Fitted Curve of Absolute Roll Angle**



**Figure 9. The Fitted Curve of Absolute Yaw Angle**



**Figure 10 The Stitching Image After the Accumulated Error Correction by the Proposed Approach**

## 5. Conclusion

The commonly used strip stereo models connection need GCPs to establish error equation for adjusting the accumulated error, which is unsuitable for a large number of images in a strip. In this paper, we present a novel approach of accumulated error correction of strip stereo models connection without GCPs. The approach can adjust accumulated error by two aspects: (a). the middle image of a strip is considered as reference; (b). the accumulated error of baseline  $B$  and angles of pitch  $\varphi$ , roll  $\omega$ , yaw  $\kappa$  is adjusted by mathematical statistics. Experimental results show that the proposed approach is available to implement strip aerial triangulation in low-altitude photogrammetry, can significantly adjust the accumulated error and improve accuracy of strip stereo models connection relative to the commonly used method.

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