Comparisons Between RV, ARV and WRV

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Abstract: Realized Volatility (RV) have been widely used since it was put forward by Anderson and Bollerslev in 1998.Some scholars put forward their improved form for optimizing the quality of RV. As a result, it is possible to comparison between RV, ARV and WRV. This paper takes a contract of stock index futures in China as sample, aiming at researching the comparison of RV, ARV and WRV.

Key Words: Realized Volatility (RV), Adjusted Realized Volatility (ARV), Weighted Realized Volatility (WRV), stock index futures

1 Introduction

Realized Volatility have been widely used since it was put forward by Anderson and Bollerslev in 1998[1]. It estimates the volatility using the sum of square of returns.It is model free and don't need researchers to estimating the parameters of the model.

Some scholars found that there exist errors when using RV in some situations, so they put forward their improved forms. Xu (2004) [2] put forward "Adjusted Realized Volatility" (ARV) and Guo (2006)[3] put forward "Weighted Realized Volatility". Both authors proved their improved form is better than RV.

After that, ARV and WRV is widely used in empirical analysis, while there isn't a literature does comparison of RV,ARV and WRV.

This paper takes a contract of stock index futures in China as sample, aiming at researching the comparison of RV, ARV and WRV.

2 Realized Volatility

Assume

$$r_{t,n} = p_{t,n} - p_{t,n-1} (t = 1,2,\cdots T, n = 1,2\cdots N)$$

$p_{t,n}$ is n th logarithmic price in t th day, $r_{t,n}$ is the return of logarithmic price of financial asset, N is the number of sample taken from [t-1,t] According to Anderson and Bollerslev(1998) [1],realized volatility is the sum of square of returns in a trading day, so

$$RV_t = \sum_{n=1}^{N} r_{t,n}^2 \tag{1}$$

3 Adjusted Realized Volatility

Xu (2004)[2] put forward "Adjusted Realized Volatility (ARV)"to dealing with the measure error of RV. Xu took the following formula as ARV：

$$ARV_t = \frac{E(RV_t) \times E(RQ_t)/n}{Var(RV_t)} + \frac{Var(RV_t) - E(RQ_t)/n}{Var(RV_t)} \times RV_t \tag{2}$$

Where: $RQ_t \equiv \frac{2}{3} n \sum_{i=1}^{n} r(t-1+\frac{j}{n},\frac{1}{n})^4$

Xu (2004) [2] proved the mean of ARV is equal to RV and the variance of ARV is smaller than RV. So in his opinion, ARV is more effective than RV.

4 Weighted Realized Volatility

According to the definition of RV, it gives an equal weight 1 to each square of intraday return. But there exists "Calendar Effects" in stock market which means the intraday return was higher in the opening and closing period which smaller in the mid-time of trading day. So it's unreasonable to imposing the identical weights. Guo (2006)[3] took "Calendar effects" into consideration and put forward "Weighted Realized Volatility (WRV)" ,It aimed at to describing the "Calendar Effects" better.

Definition 1

Weighted Realized Volatility (WRV) is a weighed sum of the square of intraday return of financial assets

$$WRV_t = \sum_{n=1}^{N} w_n r_{t,n}^2 \tag{3}$$

w_n is the weight of square of intraday returns, it meets

$$\sum_{n=1}^{N} w_n = 1 \tag{4}$$

The most important work to computing WRV is to computing the weights. Guo put forward 2 conditions :

(1)WRV is the unbiased estimator of IV

(2)WRV is the estimator with least variance.

In computing of condition 2, Guo brought Lagrange function to making the least variance, the result is

$$w_n = \frac{\sum_t \sum_{n=1}^{N} r_{t,n}^2}{N \sum_t r_{t,n}^2} \quad (5)$$

5 The comparison of statistical information among RV, ARV and WRV

5.1 Data Description

The sample came from a contract of stock index futures market of China. It ranged from Apr.16,2010-Aug 23,2011 which involved in 330 trading day.The analysis aims at the daily realized volatility of IF01,we compute it with the data of 5-minute index price.

5.2 Statistical features of RV, ARV and WRV

(1) The series of RV, ARV and WRV
Table 1 shows the statistical information of 3 original series.

From table 1, it is easy to realizing that 3 original series are all "skew to right with high peak" which meets the first item in the character of RV; their mean is equal which is consistent with the conclusion of Xu (2004) and Guo (2006).

Table 1. Statistical information of RV, ARV and WRV

	RV	ARV	WRV
Mean	0.0000911	0.0000911	0.0000911
Std.Dev	0.0000832	0.0000829	0.0000315
skewness	2.544017	2.544158	1.088602
Kurtosis	12.32425	12.32501	4.380817
Jarque-Bera	1551.408	1551.643	91.39455

(2) The series of ln RV, ln ARV and ln WRV
Table 2 shows the statistical information of 3 logarithmic series.

Table 2. Statistical information of ln RV,ln ARV and ln WRV

	ln RV	ln ARV	ln WRV
Mean	-9.620550	-9.617553	-9.358426
Std.Dev	0.792435	0.787634	0.331620
skewness	0.121150	0.133055	0.092899
Kurtosis	2.720391	2.714764	3.128948
Jarque-Bera	1.882238	2.092401	0.703292

From table 2, it's clear that 3 logarithmic series improve their result obviously in skewness, kurtosis and J-B statistics. They are much approximate to normal distribution than their original series which meets the second item of character of RV.

(3) The series of standardized returns

Set $r_t / \sqrt{RV_t}$ as standardized returns, so as to ARV and WRV. Table 3 shows the statistical information of three series of standardized returns.

Table 3. Table captions should always be positioned above the tables

	$r_t / \sqrt{RV_t}$	$r_t / \sqrt{ARV}$	$r_t / \sqrt{WRV}$
Mean	-0.036628	-0.036461	-0.068528
Std.Dev	1.558360	1.557153	1.753844
skewness	0.046757	0.046388	-0.415055
Kurtosis	3.138024	3.143562	6.280684
Jarque-Bera	0.382187	0.401738	157.4628

From table 3, it is easily to getting the following conclusion: the series of $r_t / \sqrt{RV_t}$ and $r_t / \sqrt{ARV}$ is close enough to normal distribution while the series of $r_t / \sqrt{WRV}$ features with "high peak and skew to left".

5.3 The Autocorrelation of RV, ARV and WRV

This part researches the autocorrelation of RV, ARV and WRV. Image 1-3 shows the series of 100-order autocorrelation coefficients.

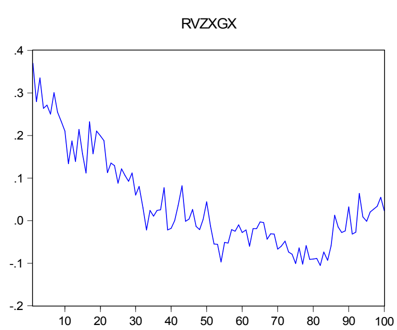

Fig. 1. 100-order autocorrelation coefficients of RV

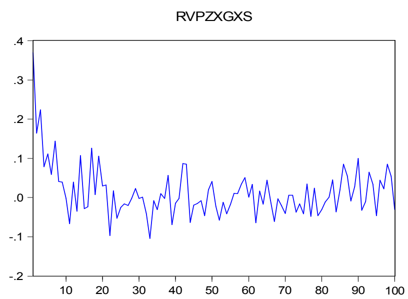

Fig. 2. 100-order partial autocorrelation coefficients of RV

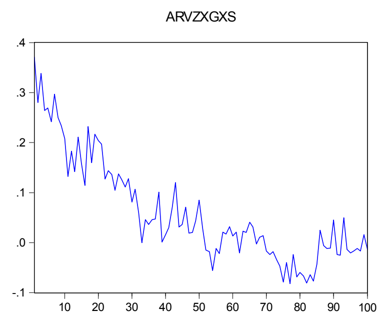

Fig. 3. 100-order autocorrelation coefficients of ARV

From Figure 1-3, all of three series have character of obvious autocorrelation which meets the first item in character of RV. The trend of autocorrelation coefficients between RV and ARV almost the same While the trend of WRV's is a little different from them. The reason is obvious: the formula of ARV can be divided into 2 parts, the first part is a product of a constant and RV, the second part is a constant. As a result, the trends of autocorrelation coefficients between RV and ARV are almost the same. But the fomula of WRV break out the structure of RV, so its trend is different from RV and ARV.

5.4 The Summary of Statistical Information of RV,ARV and WRV

Here make a conclusion about the comparison of RV,ARV and WRV
(1)Both three original series are all "skew to right with high peak" while their mean is equal;

(2)Three logarithmic series improve their result obviously in skewness, kurtosis and J-B statistics. They are much approximate to normal distribution than their original series;
(3)The three series of standard deviation are all "skew to right with high peak", but perform better than original series;
(4)The trend of autocorrelation and partial autocorrelation coefficients between RV and ARV almost the same While the trend of WRV's is a little different from them.

6 Conclusions

Realized Volatility (RV) has been widely used since it was put forward. Some scholars put forward the improved form of RV aiming at better performance in describing the volatility. This paper select 2 main form of inprovements: Adjusted Realized Volatility (ARV) and Weighted Realized Volatility (WRV) and make a comparison of RV, ARV and WRV of a contract of stock index of futures in China. The empirical analysis includes statistical information and the autocorrelation series.

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