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Optimal Linear Filter for Multi-rate System

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Abstract. This paper is concerned with the optimal linear filtering problem for system with multiple rates. The state is updated at the higher rate and the measurement is sampled at the lower rate. The least square filter is proposed by an innovation analysis approach. Compared with the existing augmented filter, the proposed filter can reduced the computational cost.

Keywords: Multi-rate; Innovation analysis approach; Filter.

1 Introduction

In recent years, the filtering problem for systems with multiple rates has attracted lots of attention due to the impossibility to sample all the physical signals at one single rate in many complicated practical systems. For multi-rate systems, the first important study goes back to the switch decomposition technique proposed by Kranc [1]. Generally, there are two methods for the state estimation problem for multi-rate systems. One is based on multiscale system theory and the other is based on Kalman filtering theory. On the basis of Kalman filtering theory, many useful filtering strategies are proposed [2-6]. But, the computational cost of the above filtering strategies is high since they are given by state/measurement augmentation. In this article, the multi-rate filtering problem is transformed into an equivalent single rate filtering problem. Then, the optimal linear filter is proposed. The proposed filter can reduce the computational cost since the state augmentation is avoided.

2 Problem Formation

Consider the following linear discrete-time stochastic multi-rate system

$$x(tb+b) = \Phi x(tb) + \Gamma w(tb) \tag{1}$$

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$$z(tb_0) = Hx(tb_0) + v(tb_0)\ , \quad b_0 = cb \tag{2}$$

where $x(tb) \in \mathbb{R}^n$ is the state vector, $z(tb_0) \in \mathbb{R}^{n_0}$ is the measured output, Φ, Γ and H are the constant matrices with suitable dimensions. $w(tb) \in \mathbb{R}^q$ and $v(tb_0) \in \mathbb{R}^{n_0}$ are white noises. The state $x(tb)$ is updated at a higher rate with a period b and the measurement $z(tb_0) \in \mathbb{R}^{n_0}$ is sampled at a lower rate with a period $b_0 = cb$ where c is a positive integer.

Assumption 1 $w(tb)$ and $v(tb_0)$ are uncorrelated white noises with zero means and variance matrices R^w and R^v, respectively.

Assumption 2 The initial state vector $x(0)$ is uncorrelated with $w(tb)$ and $v(tb_0)$, and satisfies

$$\mathrm{E}\{x(0)\} = \mu \quad \text{and} \quad \mathrm{E}\{(x(0) - \mu)(x(0) - \mu)^{\mathrm{T}}\} = \Omega \tag{3}$$

Our aim is to find the optimal linear filter $\hat{x}(tb)$ based on the measurement information $(z(tb_0), z(tb_0 - b_0), \cdots, z(0))$.

3 Optimal Linear Filter

In this subsection, we first establish the state space model at the measurement sampling points.

$$x(tb_0 + b_0) = \Phi_0 x(tb_0) + \Gamma_0 w_0(tb_0) \tag{4}$$

$$z(tb_0) = Hx(tb_0) + v(tb_0) \tag{5}$$

where $\Phi_0 = \Phi^c, \Gamma_0 = [\Phi^{c-1}\Gamma \quad \Phi^{c-2}\Gamma \quad \cdots \quad \Gamma]$, $w_0(tb_0) = [w^{\mathrm{T}}(tb_0) \quad w^{\mathrm{T}}(tb_0 + b) \quad \cdots$. $w^{\mathrm{T}}(tb_0 + b_0 - b)]^{\mathrm{T}}$. Further we have the following statistical property

$$\mathrm{E}[w_0(tb_0)] = 0\ , \quad R_0^w = \mathrm{E}[w_0(tb_0) w_0^{\mathrm{T}}(tb_0)] = \mathrm{diag}(R^w, \cdots, R^w)_{cq \times cq} \tag{6}$$

Observe that systems (4) and (5) are transformed into the single rate systems. In the following, we will give the filter $\underline{\hat{x}}(tb_0 \mid tb_0)$ and the corresponding estimation error variance matrix $\underline{P}(tb_0 \mid tb_0)$ at the measurement sampling points by applying the classical Kalman filter.

Lemma 1[7] Under Assumptions 1 and 2, the filter for systems (4) and (5) is computed by

$$\underline{\hat{x}}(tb_0 \mid tb_0) = \underline{\hat{x}}(tb_0 \mid tb_0 - b_0) + K(tb_0)\underline{\varepsilon}(tb_0) \tag{7}$$

$$\underline{\hat{x}}(tb_0 + b_0 \mid tb_0) = \Phi_0 \underline{\hat{x}}(tb_0 \mid tb_0 - b_0) + L(tb_0)\underline{\varepsilon}(tb_0) \tag{8}$$

$$\underline{\varepsilon}(tb_0) = z(tb_0) - H\underline{\hat{x}}(tb_0 \mid tb_0 - b_0) \tag{9}$$

$$K(tb_0) = \underline{P}(tb_0 \mid tb_0 - b_0) H^{\mathrm{T}} \underline{Q}_{\varepsilon}^{-1}(tb_0) \tag{10}$$

$$L(tb_0) = \Phi_0 K(tb_0) \tag{11}$$

$$\underline{Q}_{\varepsilon}(tb_0) = H\underline{P}(tb_0 \mid tb_0 - b_0) H^{\mathrm{T}} + R^v \tag{12}$$

$$\underline{P}(tb_0 \mid tb_0) = \underline{P}(tb_0 \mid tb_0 - b_0) - K(tb_0)\underline{Q}_{\varepsilon}(tb_0)K^{\mathrm{T}}(tb_0) \tag{13}$$

$$\underline{P}(tb_0 + b_0 \mid tb_0) = (\Phi_0 - L(tb_0)H)\underline{P}(tb_0 \mid tb_0 - b_0)(\Phi_0 - L(tb_0)H)^{\mathrm{T}} + \Gamma_0 R_0^w \Gamma_0^{\mathrm{T}} + L(tb_0)R^v L^{\mathrm{T}}(tb_0) \tag{14}$$

where $\underline{\varepsilon}(tb_0)$ is the innovation sequence with variance $\underline{Q}_{\varepsilon}(tb_0)$, $K(tb_0)$ is the filtering gain, $L(tb_0)$ is the one-step predictor gain, $\underline{P}(tb_0 \mid tb_0)$ is the filtering error variance matrix, $\underline{P}(tb_0 \mid tb_0 - b_0)$ is the one-step predictor error variance matrix. The initial values are $\underline{\hat{x}}(0 \mid -1) = \mu$ and $\underline{P}(0 \mid -1) = \Omega$.

Next, we will derive the filter $\hat{x}(tb_0 - lb \mid tb_0)$, $l = 1, 2, \cdots, c$ at the state updating points by applying the filter $\underline{\hat{x}}(tb_0 \mid tb_0)$ at the measurement sampling points and the state update equation(1).

Theorem 2 Under Assumptions 1 and 2, for systems (1) and (2), the filter $\hat{x}(tb_0 - lb \mid tb_0)$, $l = 1, 2, \cdots, c$ at the state updating points are as follows

$$\hat{x}(tb_0 - lb \mid tb_0) = \Phi^{-l}[\underline{\hat{x}}(tb_0 \mid tb_0) - \sum_{k=1}^{l} \Phi^{k-1}\Gamma \hat{w}(tb_0 - kb \mid tb_0)] \tag{15}$$

$$\hat{w}(tb_0 - kb \mid tb_0) = M(tb_0 - kb \mid tb_0)\underline{\varepsilon}(tb_0), \ k = 1, 2, \cdots, l \tag{16}$$

$$M(tb_0 - kb \mid tb_0) = R^w \Gamma^{\mathrm{T}} (\Phi^{k-1})^{\mathrm{T}} H^{\mathrm{T}} \underline{Q}_{\varepsilon}^{-1}(tb_0), \quad k = 1, 2, \cdots, l \tag{17}$$

where $\underline{\varepsilon}(tb_0)$, $\underline{Q}_{\varepsilon}(tb_0)$, $\underline{\hat{x}}(tb_0 \mid tb_0)$ are given by Lemma 1.

Proof: From the iteration of (1)

$$x(tb_0) = \Phi^l x(tb_0 - lb) + \sum_{k=1}^{l} \Phi^{k-1}\Gamma w(tb_0 - kb), \quad l = 1, 2, \cdots, c \tag{18}$$

By arranging (18)

$$x(tb_0 - lb) = \Phi^{-l}[x(tb_0) - \sum_{k=1}^{l} \Phi^{k-1}\Gamma w(tb_0 - kb)] \tag{19}$$

Taking projection of both sides of (19) on the linear space $L(z(tb_0), z(tb_0 - b_0), \cdots, z(0))$, we have (15). The white noise smoother $\hat{w}(tb_0 - kb \mid tb_0)$ is obtained by the following recursive projection equation

$$\hat{w}(tb_0 - kb \mid tb_0) = \hat{w}(tb_0 - kb \mid tb_0 - b_0) + M(tb_0 - kb \mid tb_0)\underline{\varepsilon}(tb_0), \ k = 1, 2, \cdots, l \tag{20}$$

From the uncorrelation of $w(tb_0 - kb)$ and $L(z(tb_0 - b_0), \cdots, z(0))$, we have the white noise predictors

$$\hat{w}(tb_0 - kb \mid tb_0 - b_0) = 0, \quad k = 1, 2, \cdots, l \tag{21}$$

Substituting (21) into (20), we can obtain (16). The white noise smoothing gain matrices $M(tb_0 - kb \mid tb_0)$ are given by

$$M(tb_0 - kb \mid tb_0) = \mathrm{E}[w(tb_0 - kb)\underline{\varepsilon}^{\mathrm{T}}(tb_0)]\underline{Q}_{\varepsilon}^{-1}(tb_0) \tag{22}$$

Substituting (2) into (9), the innovation sequences can be rewritten as

$$\underline{\varepsilon}(tb_0) = H\underline{\tilde{x}}(tb_0 \mid tb_0 - b_0) + v(tb_0) \tag{23}$$

From $\underline{\tilde{x}}(tb_0 \mid tb_0 - b_0) = x(tb_0) - \underline{\hat{x}}(tb_0 \mid tb_0 - b_0)$, $w(tb_0 - kb) \perp \underline{\hat{x}}(tb_0 \mid tb_0 - b_0)$, (18) and Assumption 1, we have

$$\mathrm{E}[w(tb_0 - kb)\underline{\varepsilon}^{\mathrm{T}}(tb_0)] = \mathrm{E}[w(tb_0 - kb)x^{\mathrm{T}}(tb_0)]H^{\mathrm{T}} = R^w \Gamma^{\mathrm{T}} (\Phi^{k-1})^{\mathrm{T}} H^{\mathrm{T}} \tag{24}$$

Substituting (24) into (22), (17) is obtained.

4 Conclusion

In this paper, a multi-rate filtering problem for linear stochastic system is studied. The considered system includes two rates: the sate updating rate and the measurement sampling rate, where the measurement sampling rate is a multiple of state updating rate. We derive the least square filter at the state updating points by projection theory. The proposed filter has lower computational burden since the non-augmented method is applied.

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