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Modelling vibration from surface and underground railways as an evolutionary random process

M.F.M. Hussein
Department of Civil Engineering, University of Nottingham, UK

Abstract

Surface and underground railway transportations help solving traffic problems in densely populated cities. Railway trains are considered as a sustainable means of transportation due to the substantial reduction of pollution resulting from the reduction of number of cars on streets. A major disadvantage of trains is the increased levels of vibration transmitted to buildings in close proximity to railways. Vibration in buildings is caused by direct transmission of ground-borne vibration generated at the wheel-rail interface due to irregularities of wheels and tracks. Vibration can cause annoyance to people and disruption of daily activities and it can also cause malfunctioning of sensitive equipment. Modelling vibration from railways is important to understand the physics of the problem and to investigate potential solutions to reduce vibration at nearby buildings.

In this paper, ground-borne vibration from trains is modelled as evolutionary random process. An evolutionary process is used because transfer functions between the axles of a moving train and a fixed observation point vary with time. The modelling process is implemented for the calculations of vibration from trains moving on floating-slab tracks for both surface and underground railways. The transfer functions for surface and underground tracks are calculated from reported models in the literature. Vibration results are calculated and presented in the form of Power Spectral Density functions and Root Mean Square values for typical roughness spectra of rails.

Keywords: ground-borne vibration, surface and underground trains, evolutionary random process

1 Introduction

Vibration from railways is one of the main environmental concerns for buildings in close proximity to railway tracks. Vibration is generated at the wheel-rail interface and propagates to nearby buildings causing annoyance to people and malfunctioning of sensitive equipment. One of the most effective methods for reducing vibration from railways is the use of floating-slab tracks. Vibration from trains is reduced by isolating the main slab track from its bed by using resilient elements, which are normally referred to as slab bearings. Floating-slab tracks are used for both surface and underground trains.

Vibration from railways is a non-stationary process. This is because the statistical information of vibration varies with time. This variation is attributed to change of positions of loads, i.e. axles of trains, with time. Vibration from moving sources is modelled as non-stationary stochastic process for various problems in the literature, for example for calculating earthquake response of structure and runway induced response of aircraft (Fang, 1977), for vehicle-bridge vibration (Lu et al., 2009; Li et al., 2008), for traffic induced vibration (Xu and Hong, 2008), and for surface railway induced vibration (Feng et al., 2006; Lombaert and Degrande, 2009).

In this paper, vibration at the free surface is calculated due to moving trains on floating-slab tracks with continuous slab. Both surface and underground tracks are considered, see Figure 1. The novelty of this work is mainly on the modelling of vibration from underground railways as a non-stationary random process and the comparisons of the response with the one resulting from surface railways.

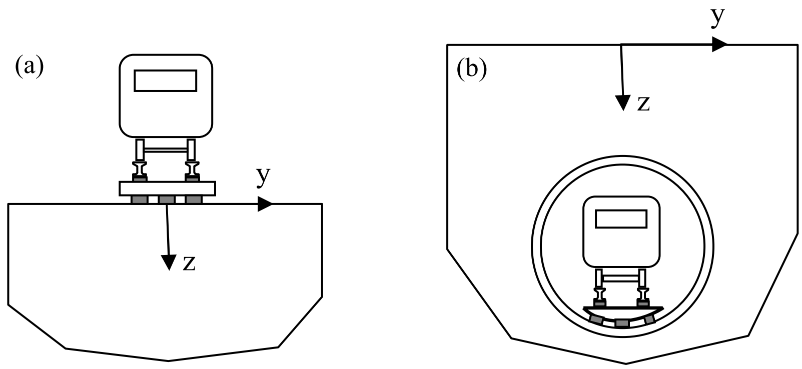

Figure 1. Vibration due to moving trains on floating-slab tracks is calculated at the free surface, i.e. at z=0, for (a) surface and (b) underground railway tracks. Resilient elements (i.e. railpads and slab bearings) are shown in dark colour

2 Modelling vibration from trains as an evolutionary random process

Vibration from trains is generated at the wheel-rail interface due to rail roughness. Rail roughness as experienced by each wheel can be modelled as a stationary random process. For a train moving with a constant velocity, vibration at an observation point in the track/soil can be considered as a stationary random process if the point is moving in the same direction and with the same velocity as the moving train. This is because the process experienced by such a point is invariant with respect to time as the position of the point is fixed with respect to the position of the train. Note that for such a case, the transfer functions between the excitation points at the wheel-rail interface and the observation point are invariant with time. Contrary to this, the position of a fixed observation point changes with the position of the train, i.e. varies with time. Note that for this case, the transfer functions between the excitation points and the observation point are dependent on time. A random process with statistical properties that varies in time can be modelled as evolutionary random process, see (Lutes and Sarkani, 2003) for example. For this paper, it is assumed that the roughness on the right and left rails are identical. It is also assumed that the roughness as experienced by different axles is uncorrelated. The effect of uncorrelation is useful as it filters out high frequency components in a Power Spectral Density (PSD) (Lombaert and Degrande, 2009). The PSD for the displacement/velocity at a fixed observation point can be calculated by

$$S_{out}(t,\omega)=\sum_{j=1}^{N}\left|H_j(t,\omega)\right|^2 \quad S_{in}(\omega) \tag{1}$$

where $H_j(t,\omega)$ is the transfer function, which gives the displacement/velocity at the observation point due to a unit harmonic roughness applied under the jth axle of the train and $S_{in}(\omega)$ is the PSD for the rail roughness experienced by each axle at an angular frequency ω . Note that

$H_j(t,\omega)$ modulates the input by giving a time-varying change for each harmonic component, resulting in a modulated process, or what is known as an evolutionary process. Using a unit value of the roughness PSD, i.e. $S_{in}(\omega)=1$, helps in investigating the general behaviour of the track-ground system. A realistic value of PSD can be calculated from the following equation (Frederich, 1984)

$$S_{in}(\omega)=\frac{1}{2\pi}\frac{av^2}{(bv+\omega/(2\pi))^3} \tag{2}$$

where v is the velocity of the train, a and b are unevenness and waviness of the track respectively and these are given in Table 1, according to the condition of the rail; whether it is in its worst, average or best condition.

Table 1. Values of unevenness a, and waviness b (Frederich, 1984)

Condition	$a\ [mm^2(1/m)^2]$	$b\ [1/m]$
Worst	9.39×10^{-1}	6.89×10^{-2}
Average	1.31×10^{-2}	2.94×10^{-2}
Best	1.90×10^{-4}	9.71×10^{-3}

3 Procedure for calculating $H_j(t,\omega)$

Vibration from a moving train is calculated by considering only the axle masses in modelling a train as the dynamics of bogies and cars are filtered by the commonly used soft primary suspensions of trains. From its definition, $H_j(t,\omega)$ can be calculated by applying a unit harmonic input under axle j of the train in the form $\Delta=e^{i\omega t}$ and eliminate $e^{i\omega t}$ from the response of the observation point, which is given by $u=H_j(t,\omega)e^{i\omega t}$. The first step in these calculations is determining the forces at the wheel-rail interface for all axles of the train, i.e. $R_{1j}e^{i\omega t}$, $R_{2j}e^{i\omega t}$…, $R_{nj}e^{i\omega t}$, where n is the total number of axles. The axles, and hence the forces, are positioned at distances x=$-L_1$, $-L_2$ …, $-L_n$ at time t=0. The computational efficiency of these forces is significantly improved, without affecting the accuracy of calculations, by assuming a weak coupling between the track and its foundation, i.e. using a model of a track on elastic foundations, see (Hussein and Hunt, 2006) for example. The value of $H_j(t,\omega)$ is then calculated by writing the equation of forces applied at rails in the wavenumber-frequency domain. This is then weighted by the transfer function between the output at the observation point and the inputs at the rails and the resulting expression is transformed back to the time-space domain. The value of $H_j(t,\omega)$ is then obtained by dividing the response by $e^{i\omega t}$ as mentioned before. It can be shown that $H_j(t,\omega)$ is given by

$$H_j(t,\omega)=\frac{1}{2\pi v}\sum_{k=1}^{n}R_{kj}\int_{-\infty}^{\infty}\widetilde{G}(\xi=\frac{\omega-\phi}{v},\phi)e^{i(\frac{\omega-\phi}{v})(L_j+x-vt)}d\phi \tag{3}$$

where $\widetilde{G}(\xi,\phi)$ is the transfer function between the response at the observation point and the loads at the rails in the wavenumber-frequency domain (ξ,ϕ). The method of calculations of the transfer function can be found in (Sheng et al., 2003, Lombaert and Degrande, 2009) for a surface railway track and (Forrest and Hunt, 2006; Hussein et al., 2006) for an underground railway track. Note that a homogeneous half-space is used for modelling the ground. The underground track is connected to the tunnel through a single line of longitudinal bearings while the surface track is connected to the free-surface through a layer of bearings and a uniform force is assumed across the layer. The integration in Eq. 3 is performed numerically using the trapezium rule and a special attention is given to the range of frequencies and Nyquist criterion, see (Stearns, 1983) for example, in order to achieve convergence.

4 Parameters and sample results

The following parameters are used to produce the results shown in the rest of this section. A train consisting of 7 cars and a total length of 111.8m (from the first to the last axle) is used for the analysis. The distance between 2 axles under one bogie is 2.3m. The distance between the back axle of a front bogie and the front axle of a back bogie is 8.8m. The distance between a back axle on a car and the front axle on the following car is 3m. The front axle of the driving car is positioned at x=0, at time t=0, see Figure 1. The train is moving in the positive x direction with a constant velocity v=40km/hr. All axles have the same mass of 1000kg.

The track consists of two rails of type UIC60 with bending stiffness=12.9 MPa.m^4 (with hysteretic loss factor of 0.02) and mass per unit length 120.6 kg/m. The track's concrete-slab has a bending stiffness of 1430 MPa. m^4 (with hysteretic loss factor of 0.05) and mass per unit length of 3500 kg/m. The stiffness of the railpads and slab bearings per unit length are 200 MN/m/m (with hysteretic loss factor of 0.15) and 5 MN/m/m (with hysteretic loss factor of 0.15) respectively. The slab bearings for the surface track are distributed over a width of 2.5m while a single line of slab bearings is modelled for the underground track.

The soil parameters are those of Oxford Clay and Middle Chalk with compression wave velocity 944 m/s, shear wave velocity 309 m/s and density 2000 kg/m^3 (with hysteretic loss factor of 0.03 associated with both pressure and shear wave velocities).

For the underground railway track, the supporting tunnel is made of concrete with compression wave velocity 5189 m/s, shear wave velocity 2774 m/s, density 2500 kg/ m^3 (with hysteretic loss factor of 0.015 associated with both pressure and shear wave velocities). The tunnel has external radius 3.0 m and internal radius 2.75 m. The distance between the tunnel centre and the free surface is 20 m.

Figure 2 shows the PSD of the vertical displacement at a point in the free surface at 40m distance from the tracks, i.e. (x=0m, y=40m, z=0m) for the surface track and (x=0m, y=32.72m, z=0m) for the underground track. Vibration is calculated for both cases due to a white noise roughness. It can be seen clearly from the figure that the track-ground system for the surface track and the track-tunnel-soil system for the underground track attenuate large frequencies and therefore an upper frequency of 150 Hz can be used for calculations of ground-borne vibration.

A close look at Figure 2 shows that large values of PSDs are concentrated at 6Hz and 80Hz. These frequencies are the slab resonance frequency, which can be simply calculated by $f_n = \sqrt{(k/M)}/(2\pi) = \sqrt{(5\times10^6/3500)}/(2\pi)$, and the mass-track resonance frequency which can be calculated using a model of a mass on a beam on elastic foundation, see (Hussein and Hunt, 2006). The power is distributed over a broader range of frequencies for the surface track due to the significant effect of Rayleigh waves. This effect also exists for the underground track, however, at low frequencies. This is because the effect of Rayleigh wave for a load applied at the surface is confined within a depth equal to the wavelength of the wave. For a source embedded in the ground, by applying reciprocity, the response in the surface will be large due to internal sources at a depth less than the wavelength of Rayleigh waves. For the current parameters of the soil, the Rayleigh wave velocity is calculated to be 292m/sec. Since the source is at depth of about 23m, this means that the effect of Rayleigh waves is small at frequencies above $f = c/\lambda = 292/23 \approx 13Hz$.

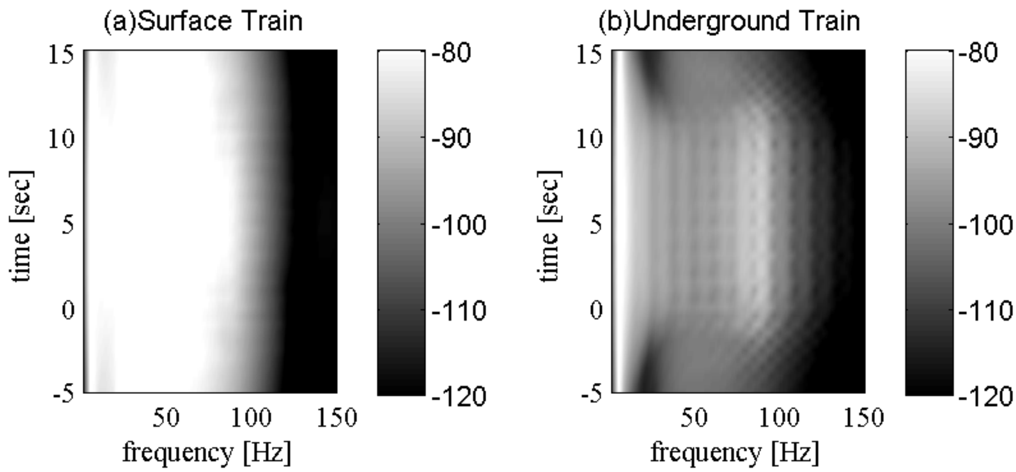

Figure 2. The PSD of the vertical displacement at a point at 40m away from tracks due to a train moving along a track with white-noise rail roughness

Figure 3 shows the vertical velocity RMS values which is calculated by integrating the results in Figure 2 after weighting by the rail roughness spectra and the square of angular frequencies to transform the displacement results to velocity. It can be seen from this figure that the RMS values for the surface train are larger than those for the underground train. There is approximately a factor of 10 increase in the vibration, for both surface and underground trains, due to the change of rail condition from best to average and from average to worst. The peak RMS value occurs approximately at the time when the distance between the centre of the train and the observation point is minimum, i.e. at time equal to the half length of the train divided by the velocity ($(111.8/2)/11.11 \approx 5\text{sec}$).

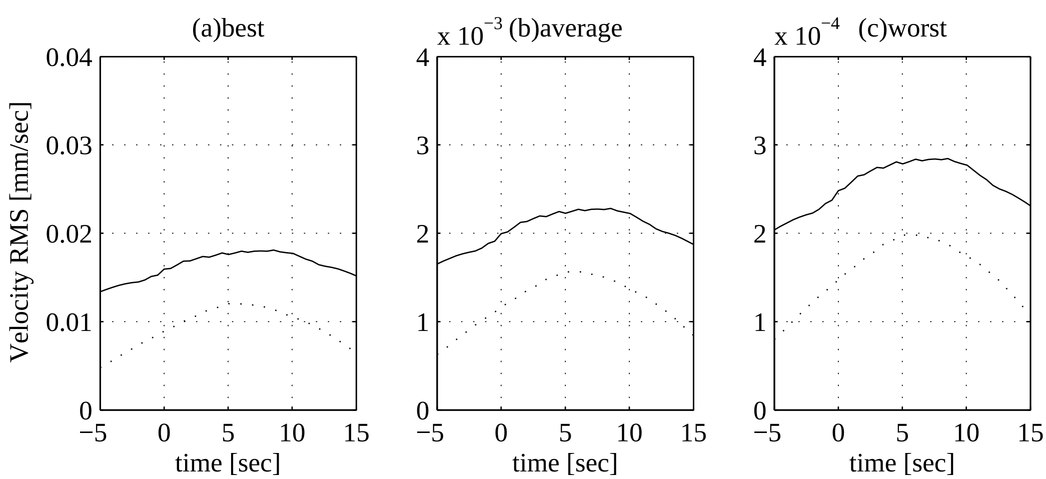

Figure 3. The vertical velocity RMS at a point at the free-surface at 40m distance from the track due to train moving on a (-) surface track and (…) underground track with a given rail roughness spectrum as described by (Frederich, 1984) for best, average and worst conditions

5 Summary and further work

Vibration from railway trains is modelled as an evolutionary random process to account for the time dependency of transfer functions due to moving trains. Floating-slab tracks for both surface and underground railways are considered. The varying PSD with time is calculated for a white noise roughness and for typical rail roughness of rails at various conditions. The PSD results presented in the paper shows large vibration at frequencies around the slab and the wheel-rail resonance frequencies. For the given parameters of the train-track-soil system for the surface track and those of the train-track-tunnel-soil system for the underground track, it is found that the peak RMS value occurs approximately at the time when the distance between the centre of the train and the observation point is minimum. It is also found that there is approximately a factor of 10 increase in the vibration, for both surface and underground trains, due to the change of rail condition from best to average and from average to worst.

The work presented in this paper is currently under further development to investigate other types of outputs at observation points. A parametric study will be carried out to understand the effect of distance from the tunnel and various parameters on vibration propagating from railway tracks. The purpose of the further work is to provide more generic conclusions.

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