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Vibration of buildings on pile groups due to railway traffic – finite-element boundary-element, approximating and prediction methods

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Abstract

A finite-element boundary-element software for the dynamic interaction of flexible structures and the soil has been extended for pile foundation. The boundary element method for the soil uses the Green's functions of the layered half-space which have been generalised for interior loads. Pile groups of 10 to 20 piles of different arrays are analysed and compared with single piles. Simplified models have been developed for a user-friendly, practice oriented prediction software for railway induced ground and building vibration.

Keywords: boundary elements, soil-structure interaction, pile foundations, prediction software

1 Combined finite-element boundary-element method

Dynamic soil-structure interaction is analysed by a combined 3-dimensional finite-element boundary-element method (AUFEBEM, Auersch, 1988). Flexible structures such as beams, plates, walls, box-type buildings, railway tracks, single piles and pile groups are modeled by the finite element method whereas the homogeneous or layered soil is modeled by the boundary element method. A point load, a wheel-set load or and a wave field have been used as excitation. In this contribution single piles and pile groups (Fig. 1) are presented.

The basis is the point load solution (Green's function) for a layered soil. The Green's functions are calculated by the following procedure. For each layer, a dynamic stiffness matrix is calculated in wave number domain (Auersch, 2008b). All layer matrices and the matrix of the underlying half-space are assembled in a stiffness matrix $\mathbf{K}_S$ of the whole soil body, which is inverted to a compliance matrix $\mathbf{N}$. The appropriate elements of this compliance matrix, for example $N_{zz}(k,z_1,z_2)$ for an interior source at z_1 and an interior response at z_2, are integrated over the horizontal wave number k to get the displacement at a horizontal distance r of the source

$$F_{zz}(r,z_1,z_2) = \frac{1}{2\pi}\int_0^\infty N_{zz}(k,z_1,z_2)\mathrm{J}_0(kr)kdk \qquad (1)$$

Similar formulas hold for the other four components of the Green's function $\mathbf{F}$. These Green's functions are used in the combined finite-element boundary-element method to establish a boundary stiffness matrix of the soil (in frequency domain, for n discrete points). A displacement matrix is compiled directly by using the point-load solutions

$$\mathbf{u}(\mathbf{x}_\beta) = \mathbf{F}(\mathbf{x}_\beta\text{-}\mathbf{x}_\alpha)\ \mathbf{p}(\mathbf{x}_\alpha) \qquad \mathbf{u}_\beta = \mathbf{F}_{\beta\alpha}\ \mathbf{p}_\alpha \qquad (2)$$

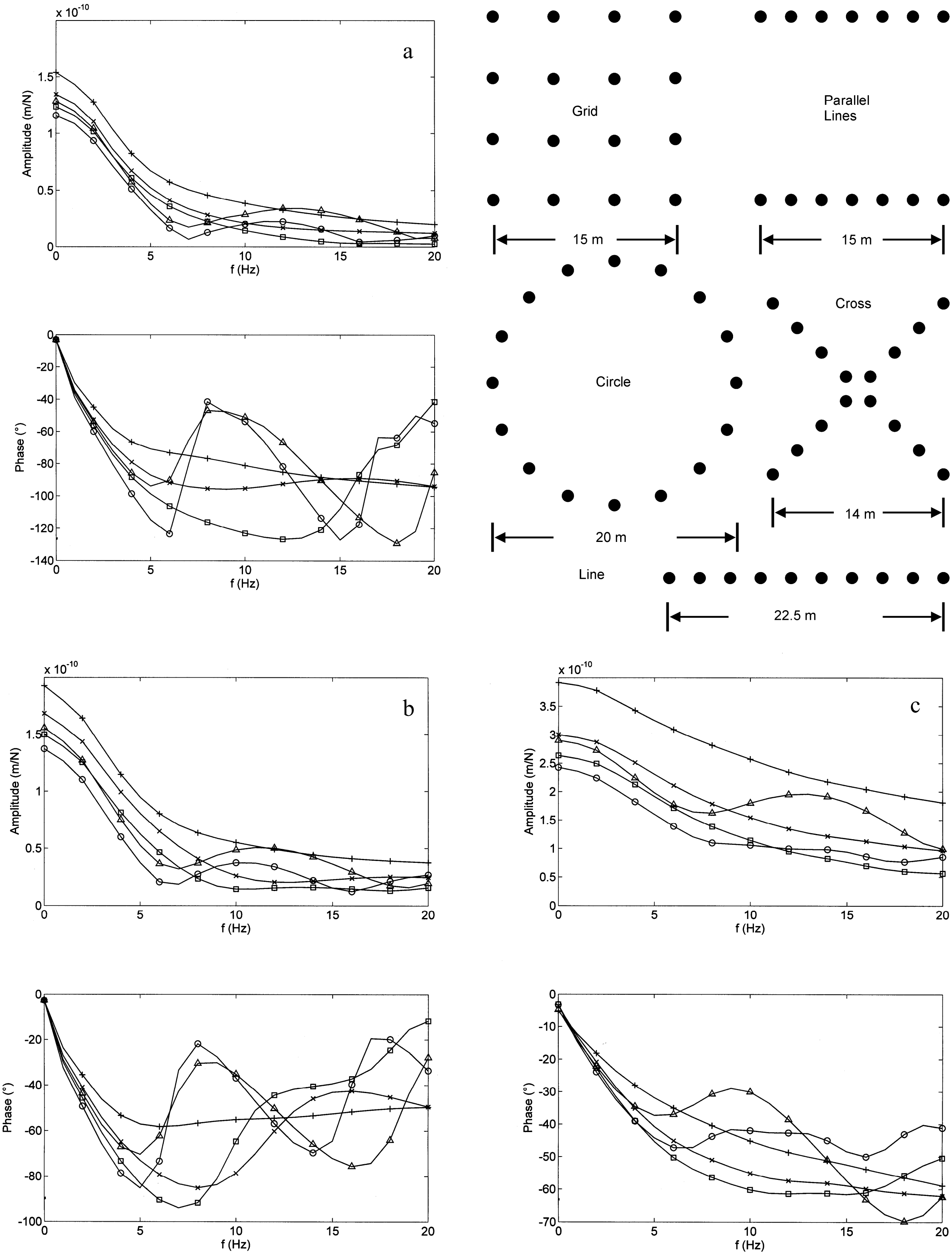

Figure 1. Amplitude and phase of the a) vertical rigid, b) vertical flexible, c) horizontal compliance of different pile groups □ grid, ○ circle, △ parallel lines, × cross and + line, concrete piles of $L = 20$ m, $R = 0.5$ m, continuum soil with shear wave velocity $v_S = 200$ m/s.

At the loading point, this would yield infinite values. Therefore, mean values on the cylindrical (in case of a pile) or circular surface (in case of a surface foundation) are calculated as the diagonal elements of the displacement matrix

$$\mathbf{u}_{\alpha} = \frac{1}{A_{\alpha}} \int_{A_{\alpha}} \mathbf{F}(\mathbf{x}-\mathbf{x}_{\alpha})\mathbf{p}(\mathbf{x}_{\alpha})\, dA \qquad \mathbf{u}_{\alpha} = \mathbf{F}_{\alpha\alpha}\, \mathbf{p}_{\alpha} \tag{3}$$

The force matrix of the soil is the identity matrix, as 1) the Green's function has no stresses at the soil surface and 2) the stresses in the interior do not yield forces at another pile element (as far as the dimension is small compared to the distance and wavelength). Thus, the inverse of the displacement matrix $\mathbf{U} = [\mathbf{F}_{\alpha\beta}]_{\alpha,\beta=1,n}$ is the boundary stiffness matrix $\mathbf{K_S}$ of the soil which is added to the finite-element stiffness matrix $\mathbf{K_B}$ of the structure to yield the global dynamic stiffness matrix $\mathbf{K}_S+\mathbf{K}_B$ of the combined structure-soil system, and the FEBE method is complete.

As an example, five different pile groups, which consist of 10 to 16 concrete piles of 20 m length and with radius of 0.5 m, are examined. The soil is characterised by its shear wave velocity $v_S = 200$ m/s ($G = 8\ 10^7$ N/m^2, $\nu = 0.33$). The frequency-dependent compliance of the pile group is given as amplitude and phase for vertical excitation of rigid (Fig. 1a) and flexible (Fig. 1b) piles, and for horizontal excitation of flexible piles (Fig. 1c) which are rigidly connected in a pile cap. At zero frequency, the horizontal compliance is more than 2-times higher than the vertical compliance. The static compliances of a single pile (3.8, 5.9 and 16 10^{-10} m/N for the vertical rigid, vertical flexible, horizontal fixed situation, not displayed) are higher than the compliance of the pile group, but not by a factor of 10 to 16. That means that a pile is more compliant in a pile group than if it is acting alone. All compliances are decreasing with increasing frequency. The phase delay is also increasing with frequency and reaches values of 60 to 130°. These effects are strongest for the vertical rigid and weakest for the horizontal flexible pile situation. The strong phase delay for the vertical rigid piles indicates a mass effect of the soil mass that is surrounded by the piles. There are some fluctuations with frequency which can be explained by waves travelling from one pile to another.

2 Approximating methods for soil-structure interaction

The results from detailed soil-structure models are compared with the results of a Winkler soil, a simple soil model which supports the foundation by spring and damper forces $P' = k'u + c'u^{\bullet}$. Under certain circumstances there is a good correlation between the exact continuum soil and the simplified Winkler results. A successful approximation has been found for strip foundations (Auersch, 1988), railway tracks (Auersch, 2006), and beams (Auersch, 2008). In case of a plate, the Winkler modulus depends on the bending stiffness of the plate (Auersch, 1996). It should be emphasized that the approximation depends on "exact" results and is restricted to certain aspects, for example the compliance of a structure-soil system. As demonstrated for railway tracks (Auersch, 2005b), for which the Winkler model is widely used without a theoretical founding, the Winkler model cannot represent all theoretical or measured effects.

In this contribution, the approximation of pile compliances is demonstrated. The corresponding "exact" results of single piles in a continuum soil are given in Figure 2 (Auersch, 2008; Lüddecke et al., 2009). The Winkler parameters are chosen and modified according to the rules which have been found for beams on the soil (Auersch, 2008). Although this has not been optimised yet, it works quite well representing results of the time consuming FEBEM calculation (Fig. 2) by a simple (explicit) and fast approximating model (Fig. 3).

At first, the vertical problem should be examined separately for finite and infinite piles. The compliance is considerably higher and the phase delay is stronger for the finite piles. The vertical Winkler solution is therefore given for finite piles in Figure 3a. The situation is completely different for the horizontal and rocking problem (Fig. 3b-d), where the loaded pile head behaves similar for a

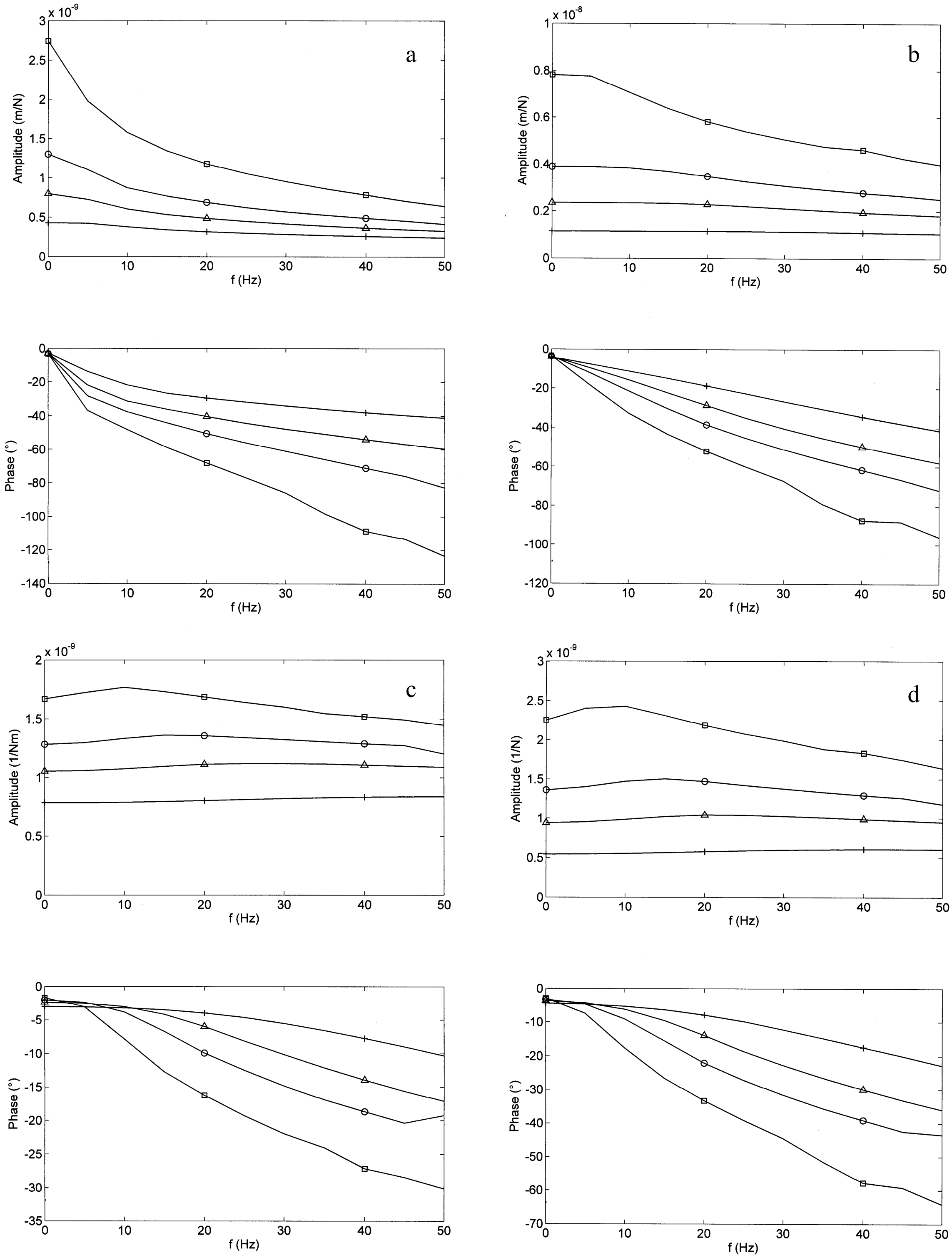

Figure 2. Amplitude and phase of the a) vertical, b) horizontal, c) rocking, and d) coupling compliance of concrete piles (L = 10 m, R = 0.5 m) on different continuum soils with v_S = □ 100, ○ 150, △ 200, × 300m/s.

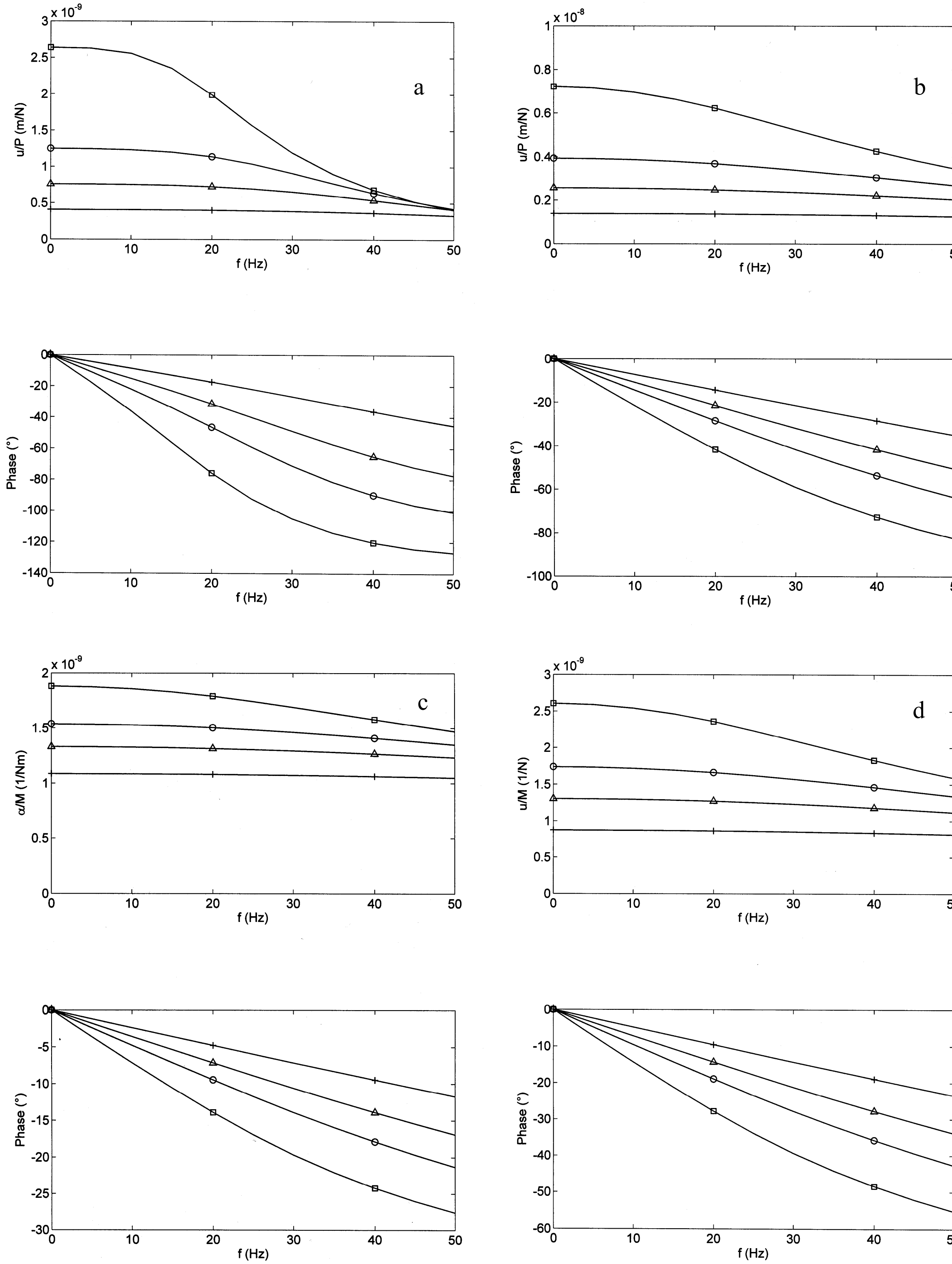

Figure 3. Amplitude and phase of the a) vertical, b) horizontal, c) rocking, and d) coupling compliance of infinite concrete piles (R = 0.5 m) on different Winkler soils with k' = □ 1, ○ 2.25, △ 4, × 9 10^8 N/m^2 and c' = □ 0.8, ○ 1.2, △ 1.6, × 2.4 10^6 Ns/m, a) vertical finite pile of L = 10 m, and with lower Winkler parameters $k' = 2G$, $c' = 4\rho v_S R$.

finite and an infinite pile. The three compliances of the infinite horizontal-rocking problem of Figure 3b-c can be given explicitly as

$$\begin{pmatrix} u \\ \alpha \end{pmatrix} = \begin{pmatrix} \sqrt{2}EI^{-1/4}k'_D{}^{-3/4} & EI^{-1/2}k'_D{}^{-1/2} \\ EI^{-1/2}k'_D{}^{-1/2} & \sqrt{2}EI^{-3/4}k'_D{}^{-1/4} \end{pmatrix} \begin{pmatrix} P \\ M \end{pmatrix} \quad (4)$$

with the dynamic support parameter $k'_D = k' + c'i\omega - m'\omega^2$.

The influence of the soil, which is presented in Figures 2 and 3, can also be derived from this formula. The strongest influence is found for the horizontal compliance ($u/P \sim k'_D{}^{-3/4}$), the weakest influence of the soil is on the rocking compliance ($\alpha/M \sim k'_D{}^{-1/4}$). The frequency dependency is ruled by the same laws, and the phase asymptotes can be obtained as $\varphi = -135°$ for the horizontal, $\varphi = -90°$ for the coupling, and $\varphi = -45°$ for the rocking compliance. All these phenomena are in good agreement between the exact and approximating model.

3 Prediction software for railway induced ground and building vibrations

The simple approximating models are integrated in a user-friendly, practice-oriented prediction software (Gerstberger et al., 2006), which yields a quick response, and needs only a minimum input easily obtainable for specialists and non-specialists. The properly coordinated modules for emission, transmission, and immission include the multi-beam track model on Winkler soil (Auersch, 2006), the layered dispersal soil model (Auersch, 2005), and the soil-wall-floor model of a building (Auersch, 2009). To conclude, computational models should be as detailed as necessary, if the correct soil-structure interaction has to be evaluated, and as simple as possible, if the observed rules have to be implemented in a prediction tool for railway induced building vibrations.

References

AUERSCH, L, 1988. *Wechselwirkung starrer und flexibler Strukturen mit dem Baugrund insbesondere bei Anregung durch Bodenerschütterungen*. BAM-Research Report 151/Thesis, Berlin.

AUERSCH, L, 1996. Dynamic plate - soil interaction - finite and infinite, flexible and rigid plates on homogeneous, layered or Winkler soil. *Soil Dynamics and Earthquake Engineering* 15, 51-59.

AUERSCH, L., 2005a. Simplified methods for wave propagation and soil-structure interaction: The dispersion of layered soil and the approximation of FEBEM results. *Proc. 6th Int. Conf. on Structural Dynamics* (EURODYN 2005), Millpress, Rotterdam, pp. 1303-1309.

AUERSCH, L., 2005b. Dynamics of the railway track and the underlying soil: the boundary-element solution, theoretical results and their experimental verification. *Vehicle System Dynamics* 43, 671-695.

AUERSCH, L., 2006. 2- and 3-dimensional methods for the assessment of ballast mats, ballast plates and other isolators of railway vibration. *International Journal of Acoustic and Vibration* 11, 167-176.

AUERSCH, L.; 2008a. Dynamic interaction of various beams with the underlying soil – finite and infinite, half-space and Winkler models. *European J. Mechanics A/Solids* 27, 933-958.

AUERSCH, L., 2008b. Static and dynamic soil-structure interaction of finite and infinite beams and plates, tracks and piles. *Proc. 7th Int. Conf. on Structural Dynamics* (EURODYN 2008), Southampton, pp. 1-12 (CD-ROM).

AUERSCH, L., 2009. Building response at foundation, walls, columns and floors due to ground vibration – simplified calculation based on experience with detailed models and measurements. *Proc. 16th Int. Congress on Sound and Vibration*, Krakow, pp. 1-8 (CD-ROM).

GERSTBERGER, U., AUERSCH, L., MEINHARDT, C., RÜCKER, W., 2006. Ein einfach handhabbares Prognose-programm für Schienenverkehrserschütterungen. *VDI-Berichte 1941 Baudynamik*, VDI Verlag, Düsseldorf, pp. 19-28.

LÜDDECKE, F., AUERSCH, L., BAESSLER, M., RÜCKER, W., 2009. Dynamisches Verhalten von Gründungen von Offshore-Windkraftanlagen. *VDI Berichte 2063 Baudynamik*, VDI-Verlag, Düsseldorf, pp. 467-480.