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Simple, yet puzzling, problems in vibration

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Abstract

Mechanical Vibration is taught to students as if it is a simple and elementary science. The single mass-on-a-spring, axial vibration of a prismatic rod, viscous dashpots, modal analysis, eigenvalues - these are all at the core of vibration science. But real vibrating systems just don't behave simply. There are pitfalls and puzzles in even the most ordinary cases and some of these will be demonstrated: a tuning fork; a bottle of soda; a bending beam; a turbocharger wheel. All of these things have simple yet puzzling behaviour. This talk will be filled with practical demonstrations - seeing is believing. Most are demonstrations that can be repeated at home.

Keywords: vibration, modal analysis, damping, Helmholtz resonator

1 Introduction

This paper is about simple problems in mechanics, but even the simplest problems can appear difficult. This is often because the observed behaviour is not what we are expecting – *i.e.* it is counterintuitive. There are several routes to counterintuitivity and something is counterintuitive if:
1) it requires advanced/specialist knowledge; 2) it is obscure or difficult to observe; 3) it doesn't fit with our experience; 4) we've never noticed it before; 5) we believed what our teachers said.

In this paper, and more particularly in the delivered presentation, many counterintuitive problems will be discussed. All of them will be demonstrated "live" and the equipment required will be minimal – almost everything is available inside the average home, office or school. Most of these demonstrations have arisen from many years of lecturing, supervising and demonstrating in dynamics and vibration at the Department of Engineering in Cambridge University. The importance of "seeing is believing" is one thing, but equally important in any lecture is the "theatre" of having things actually happening in front of the audience. It is curious that the subject area of *vibration* can yield counterintuitive problems even though it is almost entirely described by linear analysis.

2 Vibration problems

> *"In problems relating to vibrations, nature has provided us with a range of mysteries which for their elucidation require the exercise of a certain amount of mathematical dexterity. In many directions of engineering practice, that vague commodity known as common sense will carry one a long way, but no ordinary mortal is endowed with an inborn instinct for vibrations; mechanical vibrations in general are too rapid for the*

utilization of our sense of sight, and common sense applied to these phenomena is too common to be other than a source of danger."
Professor C E Inglis, FRS, James Forrest Lecture, 1944

In this quotation Inglis illustrates clearly just how counterintuive problems in vibration can be. Inglis was an engineer and these days we might translate his thinking in terms of the design process which is illustrated in Figure 1. The difficulty with dealing with vibration problems is that they are indeed mysterious and it is usual for vibration problems to be dealt with when they occur, rather than as part of the iterative design process. This keeps vibration consultants in business.

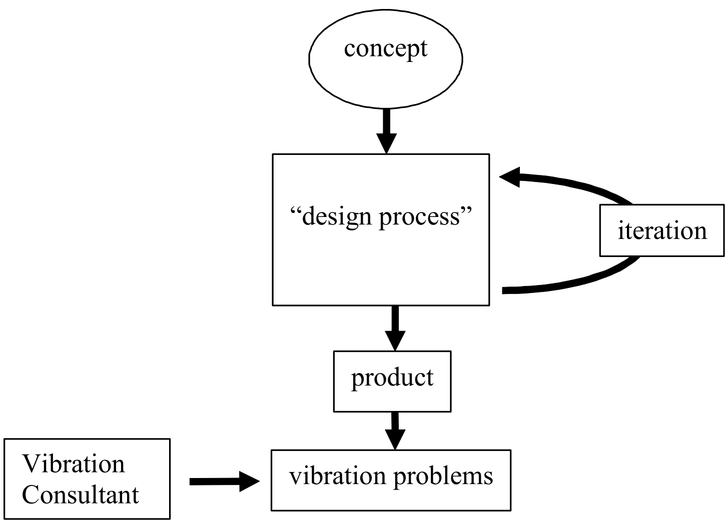


Figure 1, The design process, where vibration is usually dealt with as an afterthought.

2.1 Mass-spring-damper

The "bread and butter" of vibration analysis is the mass-on-a-spring. This is depicted in Figure 2. It is easy to understand, even without any understanding of the underlying mathematics. All an engineer needs to know is that vibration problems occur at resonance, *i.e.* when the frequency of excitation coincides with the natural frequency = $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (even the factor 2π is something we have to live with –

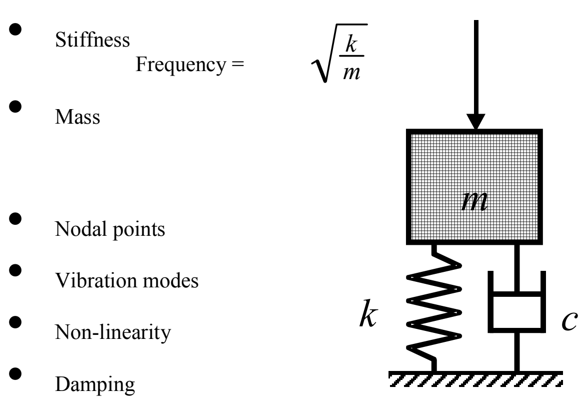


Figure 2, the mass-on-spring model, the workhorse of vibration analysis the mathematics behind it is not intuitive). So it is usual to consider raising or lowering the natural frequency by adjusting the system mass or stiffness. And if it is not possible to shift a resonance away from the frequency range of interest then it is sensible to add some damping. But there are many puzzling things that can happen, and these require an understanding of vibration modes and nodal points, and a bit about non-linearity. As for damping, the pitfalls are legion. We are lured into a false sense of security because we understand how the mass-on-spring system works.

3 The Helmholtz Resonator

A very nice experiment can be performed with an empty plastic drinks bottle, as illustrated in Figure 3. It is widely presumed that the note obtained when blowing over the open neck of a bottle is due to the "organ-pipe" resonance of the column of air within the bottle, and this explains why the not rises when water is added to the bottle. But why does the note not change if the bottle is tilted – the water moves and the air-column length changes. A little knowledge is a dangerous ting, because it is not the organ-pipe resonance at all, and in fact the simple mass-on-spring is the best explanation. The air in the neck (known as the "neck plug") acts as a mass which oscillates on the volume of air in the bottle which acts as a spring. Adding water reduces the volume of air and so stiffens the spring. This effect can be illustrated in a most interesting way by squeezing the bottle so that the air volume reduces – but very oddly the pitch of the note falls. This is because the walls of the bottle are no longer in the form of a cylinder (known for its stiffness) and the flat sides are free to vibrate. The effective stiffness of the contained air is reduced, and so the note goes down – as would be expected from $\sqrt{\frac{k}{m}}$. If the whole bottle is submerged in water, so as to stiffen the walls, the note goes up to where it might be expected to be. Throughout this entire experiment the neck-plug mass remains constant and it is only the stiffness of the contained air that changes. If the neck is lengthened then the note will go down on account of the increased neck-plug mass.

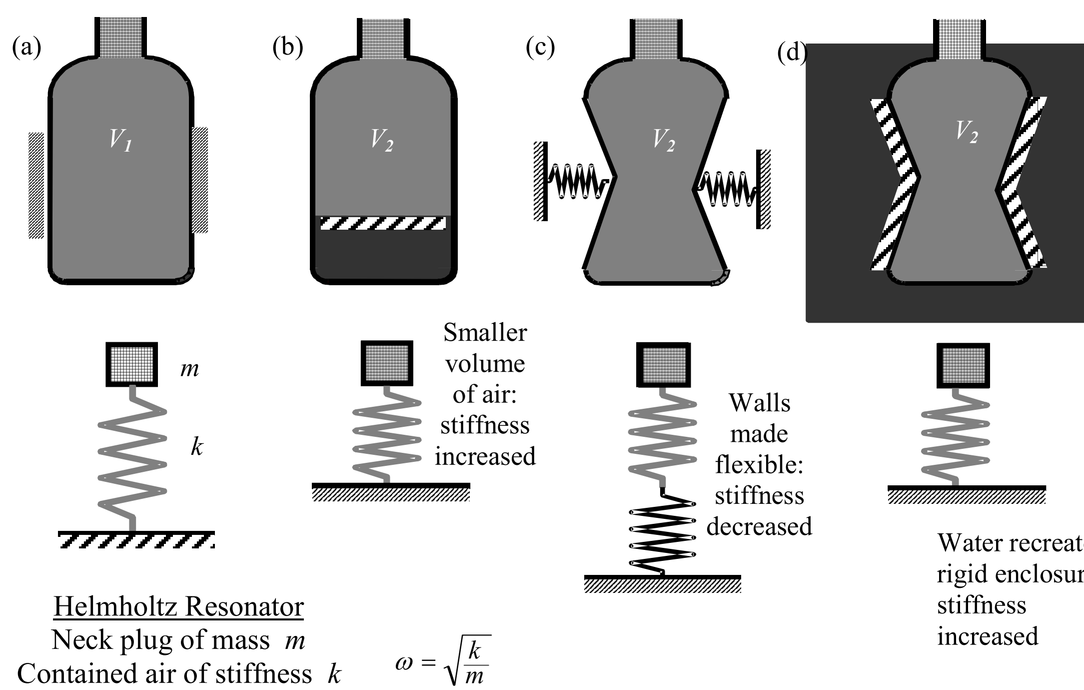


Figure 3, an empty and partly-empty plastic drinks bottle (a) Blow a note across the neck of an empty bottle. (b) add some water to reduce the bottle volume and the note goes up. (c) instead of adding water try reducing volume by squeezing the bottle – this time the note goes down (counterintuitively). (d) immerse the bottle in water and the note goes up again. All very unexpected.

4 Nodal points, non-linearities and damping

It is important to understand that very few problems can be fully understood based on the single mass on a single spring depicted in Figure 2. The following examples illustrate this, and they introduce the concepts of vibration modes, modeshapes and nodal points.

4.1 a coffee cup with a handle (nodal points)

In Figure 4 is shown a coffee mug. If it is tapped with a teaspoon then a note is heard. But listen carefully – the note is different depending on where on the rim of the cup the cup is tapped. This is because there are two possible modes that can be excited, both elliptical in shape, but one of these modes requires that the handle moves and the other has the handle at a nodal point. The moving mass will be different for the two modes and so according to $\sqrt{\frac{k}{m}}$ the frequencies will be different. This is exactly as expected but the concept of modes and nodal points has to be understood. These modes and nodal points are invisible, so as Inglis said, they are difficult to understand.

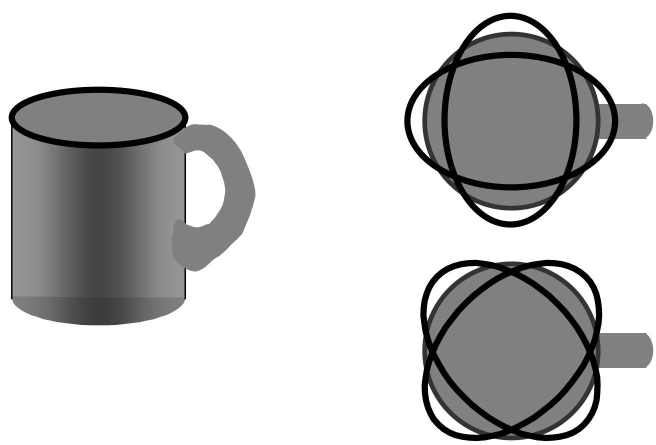
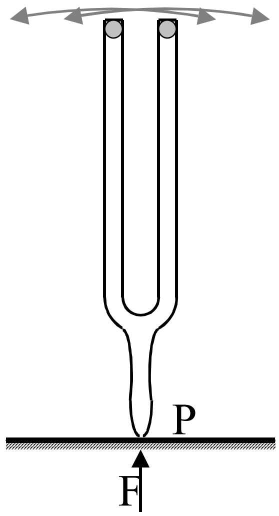


Figure 4, A coffee cup has two natural frequencies owing to the mass of the handle, so it depends where you hit it with your teaspoon.

4.2 a tuning fork (nodal points and non-linearities)

In Figure 5 is shown a tuning fork. It is well known that the mode of a tuning fork is that when the two prongs move in antiphase so that the handle os nodal. This is perfect, because the handle can be help without any danger of dissipating vibrational energy. But the conundrum is this: most musicians know that a tuning fork is too soft to be heard clearly, but that if its end is placed on a table, or on a piano lid, then the sound is much louder. But why? If the handle is nodal then surely there is no vibration there to amplify? It turns out that while the tuning fork itself is very well described by lnear vibration theory the vibration at the handle is not exactly zero. It is necessary to include the small non-linear effect due to the small arc of travel of the tips of the tuning fork. The centrifugal force causes a vertical motion and this is transmitted to the table. It is clear that this frequency must be at double the tuning fork frequency, and measurements show that it is. Our ear doesn't seem to mind an octave discrepancy in frequency – the ear seems to know how to compensate.



The tips of the tuning fork move on the arcs of circles and centrifugal inertia forces are generated, twice per cycle.

Suppose tip amplitude is 0.2mm, oscillating frequency is 440Hz, moving mass is 20% of the fork mass, then the 880Hz component of tip force F is about 10% of the weight of the fork.

Figure 5, Tuning Fork: louder when nodal point “P” is put on a table due to a small non-linear effect.

4.3 a bending beam (nodal points and mode shapes)

In Figure 6 is shown a vibrating beam. There is nothing new here but there are some very nice things to observe experimentally: 1) the beam held at different nodes produces different notes; 2) a given mode cannot be excited or damped at a nodal point; 3) all even modes are excited at the centre. The lessons from this are important. Firstly, many vibration problems can be solved by moving the source of excitation, or the observer, to a nodal point. Also, there is no point in applying vibration countermeasures to a system near a nodal point of the mode that is causing problems. All engineers should see this demonstration

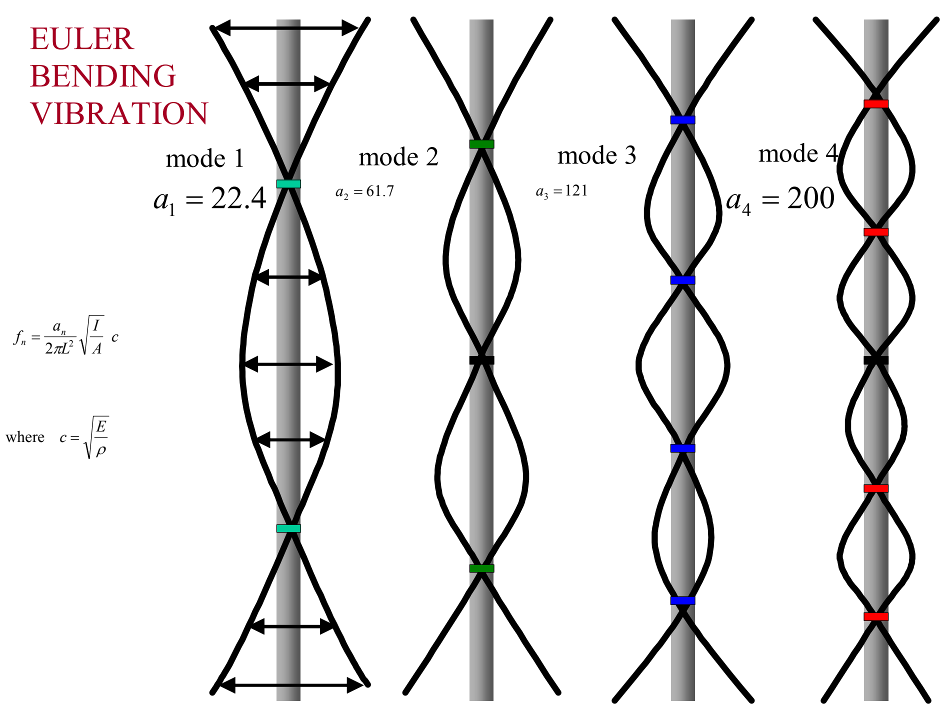
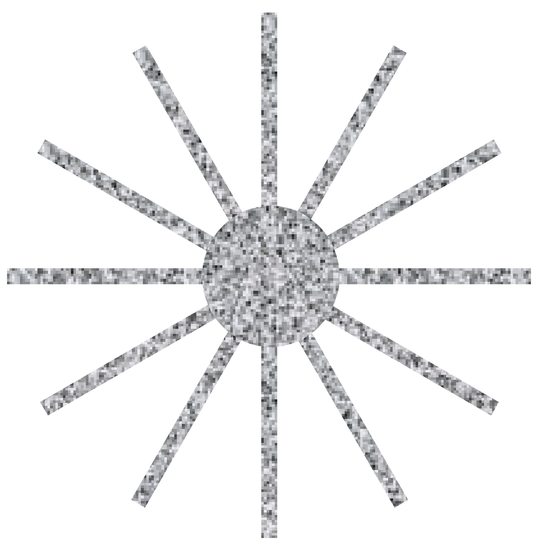


Figure 6, a vibrating beam for illustrating vibration modes and nodal points.

5 A turbocharger (nodal points, mode shapes, axisymmetry, damping

To end with a most extraordinary demonstration, consider a simple model for a turbocharger, as shown in Figure 7. According to modal analysis as observed on the bending beam of Figure 6, it would make sense to suppose that a blade that is excited by tapping it with a hammer can be silenced by holding it at the same point as it was excited. But this is not what happens. It will be shown that each of the blades can be "stopped" in turn and the turbocharger keeps on vibrating. The reason for this is that there is coupling between the various blades and that this coupling is through damping. Damping is so poorly understood at the best of times and it would be beyond the experience of almost any engineer to expect that damping is responsible for this totally counterintuitive phenomenon. It goes some way to explain why turbocharger vibration is so difficult to model and to understand.



Axisymmetric bodies

Turbocharger blade vibration

Questions:

- Do the blades fatigue less rapidly if they are perfectly tuned, or is it better to mistune them?
- Can vibration measurements made on a rotor be used to estimate its fatigue life?

Figure 7, vibration of a model turbocharger

6 Conclusions

The bottom line of all this is that dynamics and vibration are both very interesting subjects of study, and they impinge on many engineering problems. But without seeing first hand how some really simple problems can be totally baffling one might be led to believe that all problems in dynamics and vibration have simple solutions. This is certainly not the case. I recommend that all engineers try some of these simple experiments for themselves – it is a humbling experience.