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Simulating ground vibration from underground railways through subsiding soil layers

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Abstract

Underground railways are a proven method for effectively transporting people in densely populated areas. Unfortunately, ground-borne vibration from these underground railways is a major source of disturbance for individuals either working or living near subway tunnels. Numerical models used to predict vibration from these subways commonly use simplifying assumptions for the soil to reduce computational complexity: homogeneous soil properties, horizontal layering, etc. The level of uncertainty which results from these assumptions is not widely understood. For instance, satellite interferometric techniques show significant levels of subsidence can develop over railway tunnels. This paper investigates the effect of subsiding soil layers over underground railways on surface vibration compared to horizontal soil layering. The authors' coupled thin-layer/pipe-in-pipe plane-strain model is extended to account for subsiding layers of varying degree and material properties. The model predicts that a realistic level of subsidence results in minor changes to surface vibration (1.5 dB rms particle velocity). This suggests that neglecting to account for subsidence in computational models of vibrations from underground railways is a reasonable assumption.

Keywords: underground railway, thin-layer method, vibration, subsidence, soil

1 Introduction

Underground railways can efficiently transport large numbers of people in densely populated areas: per-capita power consumption and pollution emissions are low compared to personal transport and the majority of subway infrastructure is below ground eliminating traffic congestion in urban centres. However, studies show that individuals subjected to noise and vibration from rail traffic report high levels of annoyance and sleep disturbance (Fidell et al., 1991; Miedema and Vos, 1998). The frequency range of interest is between 5Hz and 200Hz as listed in BS 6472:1992 and BS ISO 14837 Part 1. Predictive models should focus on the problem frequency range between 15Hz and 150Hz; higher frequencies are attenuated by the soil through geometric decay and material damping, while lower frequencies are weakly excited and generally below the threshold of human perception (Greer and Manning, 1998). The vibrations of interest arise principally from the quasi-static load of the train moving along the track, general wheel and rail unevenness, and periodic changes in rail-support stiffness when sleepers are present (Forrest, 1999).

Designers of underground railways and surrounding buildings can use vibration predictions to allow for efficient and economical planning of vibration countermeasures; finite element (FE) and boundary element (BE) methods are commonly employed for such simulations. Although FE/BE permit accurate modelling of complex geometrical regions (e.g. square tunnels, piled foundations, etc.) these methods suffer from two major drawbacks: input accuracy and speed. The dynamic properties of soils are difficult to measure and can vary significantly over the area of interest (Hunt, 1988) thus it is difficult to determine the correct input parameters for the model. As such it is beneficial to perform parametric studies to cover a range of possible soil parameters; however FE/BE models of underground railways can require tens of hours to compute the response at a single loading frequency making in-depth parametric studies intractable.

A more economical approach to simulating ground vibration due to underground railways involves semi-analytical methods. Previous papers by the authors (Jones and Hunt, 2008; 2009) detail a computationally efficient, semi-analytical formulation which can account for various modelling uncertainties associated with soil, such as inclined soil layers and soil inhomogeneity. The current paper further develops this model to investigate the effect of soil subsidence over the tunnels on ground vibration.

2 Subsidence over tunnels

Ground movement associated with the construction of underground railway tunnels is inevitable. As the tunnelling face progresses forward the lack of support for the overburden causes the ground above the tunnel to sag, as depicted in Figure 1. This trough can be described by a Gaussian error function as shown in Figure 2 and described mathematically below (Peck, 1969; O'Reilly and New, 1982)

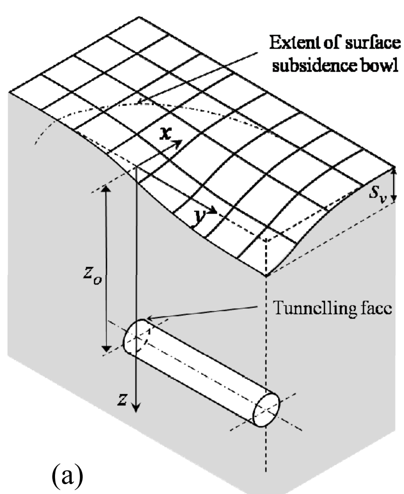
$$f(x) = S_v e^{\left[\frac{-(x-xo)^2}{2(i_x)^2}\right]} \quad (1)$$

where

$$S_v = \frac{1.25V_L}{0.175+0.325\left(1-\frac{z}{z_o}\right)}\frac{R^2}{z_o} \quad (2)$$

$$i_x = 0.5(z_o - z) \quad (3)$$

and x and z describe the location of interest for the subsidence estimation, x_o and z_o are the location of the centreline of the tunnel, V_L is the volume loss per unit length, i_x is the distance from the centre of the trough to the point of inflections (Fig. 1b), and R is the radius of the tunnel being excavated.



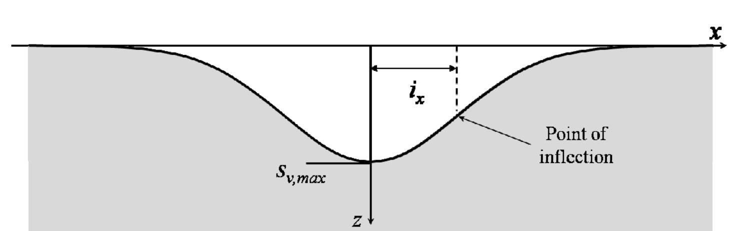


Figure 1: Schematic showing (a) the development of a surface subsidence trough as tunnelling proceeds forward, and (b) the Gaussian nature of a subsidence trough over an underground railway tunnel

O'Reilly and New (1982) list a number of volume loss estimations (V_L) from underground railway sites in the UK ranging from 1.5% to 2.5% shortly after construction. More current studies using space radar interferometric techniques show subsidence levels of $S_{v,max}$ = 10-20 mm/year (V_L = 3-9%) for underground railways in the UK, Korea, Chile and Greece (Knight, 2002; Kim et al., 2007; Parcharidis, 2006). The yearly subsidence over the tunnels is attributed to water leakage into the tunnels resulting in a loss of pore pressure in the surrounding soil. It is conceivable that a subsidence trough of 40-50mm could develop over an underground railway tunnel during its lifetime. The goal of the current paper is to quantify the effect such subsidence would have on surface vibrations.

3 Numerical Model

The current model is an extension of the previous plane-strain model used by Jones and Hunt (2008; 2009) which employs semi-analytical elements (i.e. hyperelements, semi-infinite elements, and halfspace elements) to predict ground vibration. The elements utilize the analytical solution for horizontal wave propagation while assuming vertical displacements vary linearly through the thickness of the element. The basic model is made up of four zones. The central zone is composed of hyperelements to simulate the subsidence area; on either side of this central area are semi-infinte zones extending outward modelled with semi-infinite elements. Below the central area is a final semi-infinite zone extending downward using halfspace elements. The section below provides a brief review of the modelling theory followed by a detailed description of the current model.

3.1 Semi-infinite elements

The Navier equation governing motion for homogeneous, isotropic, linear elastic bodies, is given by Graff (Graff, 1991) as

$$G\nabla^2\boldsymbol{u} + (\lambda + G)\nabla\nabla\cdot\boldsymbol{u} + \rho\boldsymbol{b} = \rho\ddot{\boldsymbol{u}} \quad (4)$$

where $\boldsymbol{u}$ is the displacement vector in the x, y, and z-directions, G and λ are Lame's constants of the solid, ρ is the density of the solid, and $\boldsymbol{b}$ is the body load vector. Waas (1972) solves Eq. 4 for plane-strain in the frequency-wavenumber domain by treating the solid as a continuum in the horizontal direction but discretising in the vertical direction, resulting in the following equation

$$(\mathbf{A}k^2 + i\mathbf{B}k + \mathbf{C})\boldsymbol{v} = 0 \quad (5)$$

where

$$\mathbf{C} = \mathbf{G} - \omega^2\mathbf{M} \quad (6)$$

The vector $\boldsymbol{v}$ contains the modal coordinates in the x and z directions for the n layers at a given wavenumber k. The $2n$ x $2n$ matrices **A**, **B** and **C** consist of the contributions from the n individual layers and are assembled using standard FEA stiffness matrix addition; the submatrices used to construct Eq. 5 are given in the appendix where the subscript j refers to the j^{th} layer.

For any given frequency, Eq. 5 has a non-trivial solution $\boldsymbol{v}$ if and only if

$$|\mathbf{A}k^2 + i\mathbf{B}k + \mathbf{C}| = 0 \quad (7)$$

This results in a quadratic-eigenvalue problem in k, the wavenumbers for the layered region. The solution to Eq. 7 consists of $4n$ eigenvalues: a set of $2n$ values with negative imaginary parts **K** and $2n$ values with positive imaginary parts $\widetilde{\mathbf{K}}$, where it can be shown $\widetilde{\mathbf{K}} = -\mathbf{K}$. The set of eigenvalues with negative imaginary parts correspond to waves traveling to the right while those with positive imaginary parts correspond to waves traveling to the left. The associated eigenvectors (i.e. modeshapes) are designated by **X** and $\widetilde{\mathbf{X}}$ respectively.

It can be shown that for a semi-infinite region, extending to the right

$$\boldsymbol{P} = \mathbf{R}\boldsymbol{U} \tag{8}$$

where

$$\mathbf{R} = i\mathbf{AXKX}^{-1} + \mathbf{D} \tag{9}$$

$\mathbf{R}$ is equivalent to the stiffness of the layered region in plane-strain, $\boldsymbol{P}$ is the external forcing vector, $\boldsymbol{U}$ is the displacement vector in the frequency domain; the definition of $\mathbf{D}$ is given in the appendix. For a full derivation refer to Waas, 1972.

This method was developed for a semi-infinite soil layer resting on rigid bedrock, however Andrade (1999) later extended this method to include a ``halfspace" element allowing for simulation of a soil layer resting on a halfspace. Furthermore, Kausel and Roësset (1977) extended this method to allow elements of finite length (i.e. hyperelements). This is accomplished by accounting for the waves travelling in both directions through the layer due to nodal loading at both edges of the element. The extensions are based on the semi-infinite formulation outlined above; for a full derivation please refer to the original references. Using hyperelements for the interior of the model, semi-infinite elements on the edges and halfspace elements on the base, a computationally efficient model of a subsiding, layered halfspace can be constructed as outlined below.

3.2 Subsiding layers model

The plane-strain example case considered in this paper is shown in Figure 3, consisting of an upper layer resting on a halfspace, a buried tunnel, and an area of subsidence over the tunnel whose geometry is described in Section 2. The surface of the soil is assume horizontal, as would be the case if a building site with associated landscaping was located above the tunnel.

The property values are listed in Table 1; these soil values are common for London Clay which is commonly found around the London Underground. A white-noise harmonic line-load is applied to the bottom of the tunnel invert for frequencies ranging between 10Hz to 150Hz with 5Hz steps. The rms particle velocity at the surface is calculated using standard random vibration theory (Newland, 1984) used in previous simulations (Jones and Hunt, 2009).

As shown in Table 1, the elastic modulus of the upper layer is varied by factors of 4 to investigate the effect on a layer with both faster and slower wave speeds than the halfspace. The extent of the subsidence is varied between $S_{v,max}$ = 0 and 67 mm and the results are compared to determine the effect of subsidence on surface vibration.

4 Results

The surface rms particle velocity for the case of a softer upper layer (E_1 = ¼*550 MPa) and for a stiffer upper layer (E_1 = 4*550 MPa) are presented in Figs. 4a and 4b, respectively. Each figure contains four curves representing the surface response for increasing levels of subsidence above the tunnel. The results from the case of a softer layer (Fig. 4a) suggest that as the level of subsidence increases the surface rms particle velocity decreases. The maximum difference between the case of a horizontal layer ($S_{v,max}$ = 0mm) and a case including subsidence is -1.4 dB which occurs for case $S_{v,max}$ = 67mm at the surface location +/-15m. The results from the case of a stiffer layer (Fig. 4b) suggest the opposite; the surface rms particle velocity increases as subsidence increases. The maximum difference between the case of a horizontal layer ($S_{v,max}$ = 0mm) and a case including subsidence is 1.6 dB which occurs for case $S_{v,max}$ = 67mm at the surface location +/-15m.

The semi-analytical simulation took an average of 11 seconds per frequency step to calculate the surface rms velocity. In comparison, a boundary element model of similar complexity used in a previous verification model (Jones and Hunt, 2009) took on average 8 minutes per frequency step.

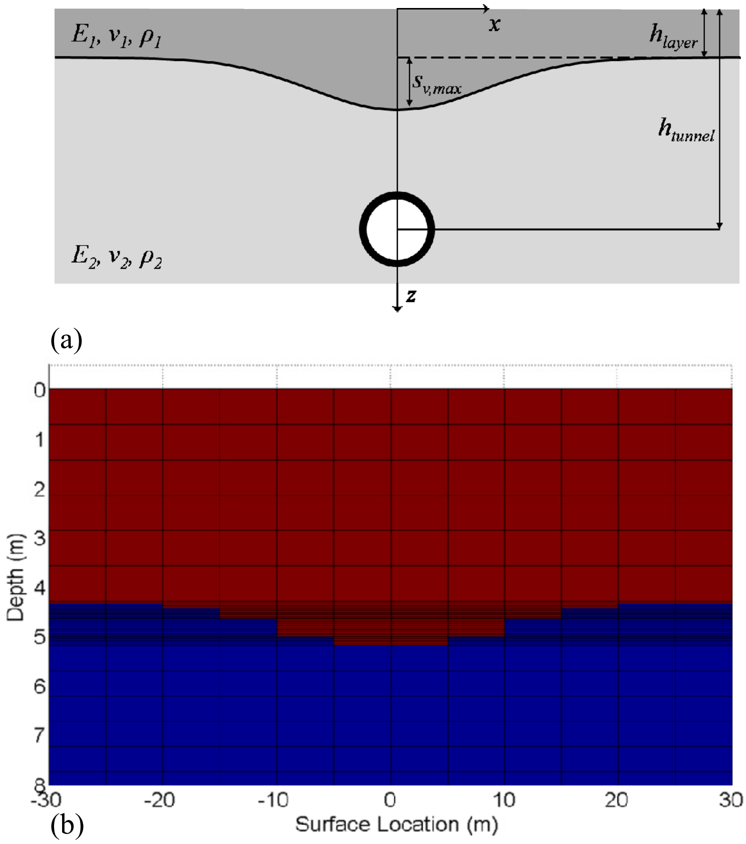


Figure 3: (a) Schematic of model showing two soil layers, tunnel and subsidence zone; (b) hyperelement model showing subsidence area (red elements signify upper layer; blue elements signify halfspace layer)

Table 1: Parameter values

Constant variables	
h_{layer}	5 m
h_{tunnel}	15 m
E_2	550*(1+0.1i) MPa
v_1, v_2	0.44
ρ_1, ρv_2	2000 kg/m^3
E_{tunnel}	50*(1+0.1i) GPa
v_{tunnel}	0.3
ρ_{tunnel}	2500 kg/m^3
Parametric variables	
E_1	(¼, 4)*E_2
$S_{v,max}$	(0, 22, 45, 67) mm

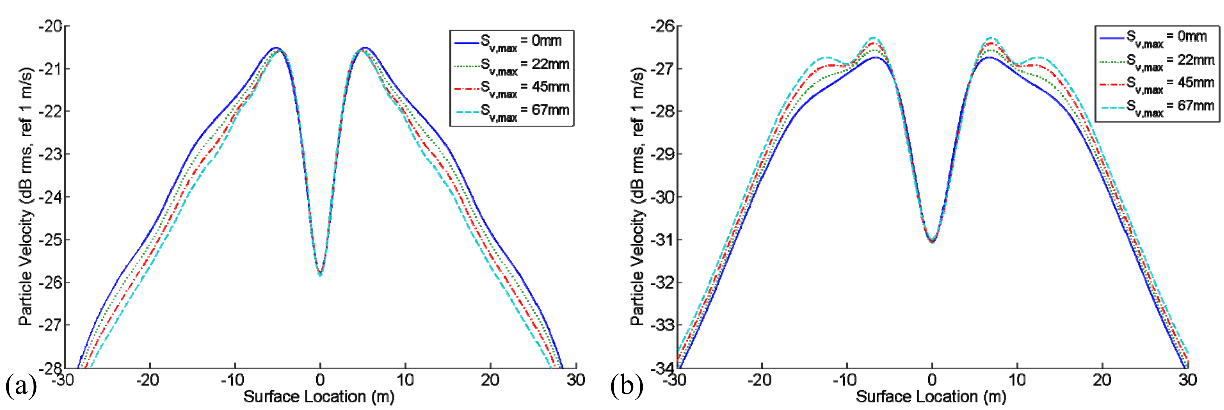


Figure 4: Surface rms particle velocity response for increasing levels of subsidence above and underground railway tunnel; (a) upper layer with softer elastic modulus (E_1 = ¼*550 MPa), and (b) upper layer with stiffer elastic modulus (E_1 = 4*550 MPa)

5 Conclusions and Future Work

Predictive modeling of ground-borne vibration due to underground railways is of growing importance in densely populated urban centers. Uncertainty associated with the assumptions used in these models must be understood to provide predictions with a reasonable level of accuracy. A semi-analytical model is presented herein which can quickly predict ground vibration in layered media which has undergone subsidence above the tunnel. The small difference in surface rms velocity predicted between the models with subsiding layers versus horizontal layers suggests that neglecting to account for subsidence when simulating ground vibrations from underground railways will not add a significant amount of uncertainty to the predicted levels (+/- 1.5 dB rms, ref 1 m/s).

The model is currently being extended to account for stochastically varying material properties due to depth or soil inhomogeneity. The Karhunen-Loeve expansion theorem is being investigated for modelling the smooth variation in properties between hyperelements.

Appendix

$$[\mathbf{A}]_j = \frac{h_j}{6}\begin{bmatrix} 2(2G_j+\lambda_j) & 0 & (2G_j+\lambda_j) & 0 \\ 0 & 2G_j & 0 & G_j \\ (2G_j+\lambda_j) & 0 & 2(2G_j+\lambda_j) & 0 \\ 0 & G_j & 0 & 2G_j \end{bmatrix} \quad [\mathbf{G}]_j = \frac{1}{h_j}\begin{bmatrix} G_j & 0 & -G_j & 0 \\ 0 & (2G_j+\lambda_j) & 0 & -(2G_j+\lambda_j) \\ -G_j & 0 & G_j & 0 \\ 0 & -(2G_j+\lambda_j) & 0 & (2G_j+\lambda_j) \end{bmatrix}$$

$$[\mathbf{D}]_j = \frac{1}{2}\begin{bmatrix} 0 & \lambda_j & 0 & -\lambda_j \\ G_j & 0 & -G_j & 0 \\ 0 & \lambda_j & 0 & -\lambda_j \\ G_j & 0 & -G_j & 0 \end{bmatrix} \quad [\mathbf{B}]_j = \frac{1}{2}\begin{bmatrix} 0 & (G_j-\lambda_j) & 0 & (G_j+\lambda_j) \\ -(G_j-\lambda_j) & 0 & (G_j+\lambda_j) & 0 \\ 0 & -(G_j+\lambda_j) & 0 & -(G_j-\lambda_j) \\ -(G_j+\lambda_j) & 0 & (G_j-\lambda_j) & 0 \end{bmatrix}$$

$$[\mathbf{M}]_j = \frac{\rho_j h_j}{2}\begin{bmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{bmatrix} \quad [\mathbf{T}] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$$

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