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Turbulence in Plasmas: What is different from Turbulence in Neutral Fluids?

In old Democrit's mind the world consisted of four elements: earth, water, air and fire. The modern view is not much different, we have four aggregate states of matter: solids, liquids, gases and plasmas. We may view these as four steps on a temperature ladder. Starting with ice, it produces water when melting, which again produces vapour – that is a gas – when boiling. Finally, the gas, if passed through a flame, is transformed into a plasma where a certain fraction of the atoms or molecules of the gas has been broken up into positively charged ions and free negatively charged electrons. The plasma is thus an ionized gas. The charged particle species on the one hand interact with any electric or magnetic field present, on the other hand they are themselves sources for such fields. This fact is responsible for a vastly different physics of plasmas compared with that of the neutral gas. In fact, the new properties are so dominating that a gas where only one atom for every 10,000 atoms is ionized is commonly denoted as a strongly ionized gas. We sometimes refer to liquids, gases or plasmas as fluids.

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The complexity of the physics increases significantly when going from a neutral fluid to a plasma. This comes as a result of the importance of the long range effects of electric and magnetic fields. Thus a local disturbance in the plasma that produces an unbalance in the charge density at some point will generate fields that will be felt by charged particles at large distance. This gives the possibility for coherent oscillations in plasmas with no counterparts in neutral fluids. With this larger selection of possible wave modes in plasmas, some of these may go unstable in the presence of a proper free energy source, leading to the generation of a turbulent state.

The free energy source may be of different types. In a neutral fluid a velocity shear is a common free energy source. The corresponding instability produced is named after Kelvin and Helmholtz, two of the giants in the history of physics. A velocity shear means that the mean flow velocity of the fluid varies in space in the span-wise direction, that is, in the direction perpendicular to the flow itself. This means that neighbouring layers of the fluid are moving with different velocities. This situation arises if a fluid is forced through a circular tube and where the flow velocity near the centre line is larger than that near the wall. A similar situation results if a fluid jet is forced into an otherwise stationary fluid. Instabilities and turbulence will result if the forcing is strong enough. The typical turbulent plumes from industrial smokestacks are common examples of this situation.

The Kelvin-Helmholtz mechanism will be acting also in an electrically conducting fluid like a plasma. In this medium, however, a large selection of alternative free energy sources are available. One example is the presence of a drift of electrons relative that of the ions. This will produce a net transport of charge in the plasma, and therefore an electric current. It is a well-known fact that at plasma carrying a threadlike current will go unstable if special precautions are not taken. This fact was one of the first stumbling blocks met with in the research toward the controlled nuclear fusion reactor.

But even a relative drift of two parts of the same particle population will do as a suitable free energy source. This situation will arise for instance when an ion beam is penetrating a stationary ion background population. To give a mathematical description of this situation it is no longer enough to represent the ion specie in terms of its mean density, its mean velocity and temperature as is commonly done with other fluids. To describe the properties and dynamics of this system it is necessary to introduce the time varying particle probability distribution function $f(\boldsymbol{r}, \boldsymbol{v}, \mathrm{t})$ in the six-dimensional phase space. The phase space $(\boldsymbol{r}, \boldsymbol{v})$ consists of the combination of the three-dimensional configuration space with the three-dimensional velocity space. The distribution function f expresses the probability of finding particles of the given species at a given position $\boldsymbol{r}$ and velocity $\boldsymbol{v}$ per unit phase space volume at the given time t. From the distribution function the particle density $n(\boldsymbol{r}, t)$ and mean drift velocity $\boldsymbol{V}(\boldsymbol{r}, t)$ of a given particle species can be found by an integration over velocity space,

Equation 1:

$$n(\boldsymbol{r}, t) = \int f(\boldsymbol{r}, \boldsymbol{v}, t)\, \mathrm{d}^3\boldsymbol{v}$$
$$n(\boldsymbol{r}, t)\, \boldsymbol{V}(\boldsymbol{r}, \mathrm{t}) = \int \boldsymbol{v}\, f(\boldsymbol{r}, \boldsymbol{v}, t)\, \mathrm{d}^3\boldsymbol{v}.$$

For a quantitative description of the plasma dynamics we will thus need an equation telling how the distribution function f evolves in phase space as a function of time. The relevant equation is conceptually and intuitively simple. The distribution function should remain constant if we move along particle trajectories in phase space. A particle at position $\boldsymbol{r}$ with velocity $\boldsymbol{v}$ at time t will at time $t + \mathrm{d}t$ have position $\boldsymbol{r} + \boldsymbol{v}\, \mathrm{d}t$ with velocity $\boldsymbol{v} + q\boldsymbol{E}(\boldsymbol{r}, t)\, \mathrm{d}t \,/\, m$. Here q and m are the charge and mass of the chosen particle species and $\boldsymbol{E}$ is the electric field present. In the last term $q\, \boldsymbol{E}(\boldsymbol{r}, t) \,/\, m$ represents the acceleration, that is, the instantaneous change in velocity that a particle at position $\boldsymbol{r}$ at time t will suffer per unit time due to the presence of the electric field. For simplicity we here neglected the corresponding effect due to magnetic fields. The constancy of the distribution function along particle trajectories is now conveniently expressed through the partial differential

Equation 2:

$$\partial f \,/\, \partial t + \boldsymbol{v} \bullet \partial f \,/\, \partial \boldsymbol{r} + q/m\, \boldsymbol{E} \bullet \partial f \,/\, \partial \boldsymbol{v} = 0.$$

This equation is, however, not quite as simple as it may look. The electric field $\boldsymbol{E}$ is again determined by the net charge density produced by all particle species present, that is, determined by velocity integrals of all evolving distribution functions. In the proper terminology, the field $\boldsymbol{E}$ to be substituted in (1) is the self-consistent electric field produced by the plasma itself in addition to possible external fields present. This fact turns (1) into a non-linear differential-integral equation in f. In addition the equation as it stands does not include effects produced by close encounters between the particles in the plasma, including possible ionization and recombinationprocesses. These additional effects can be included by adding rather complicated non-linear collision terms to the right hand side of (Equation 1).

Convenient mathematical tools are available for the analysis of linear systems. For non-linear systems the situation is different, each case requiring its special tricks, if at all within our present analytical capability. For the present problem some progress can be made by studying a simplified linearized version of (Equation 1). The very early stage of an unstable situation may be analyzed this way. For the later saturated and turbulent state this approach is, however, not sufficient. The full effect of non-linear terms has to be included.

For this reason numerical simulation techniques have become a very important tool for advancing our understanding of plasma dynamics. We will illustrate this by considering one example in the following, an example that we are presently actively studying. In this example a plasma beam, consisting of co-moving ions and electrons, is penetrating a neutral gas. For the illustration we again simplify the problem, this time not by removing any non-linearity, but by assuming that certain aspects of the problem can be described in a simplified one-dimensional model, that is, by assuming that the distribution function only depends on one spatial variable x and that the dynamics can be limited by including only one velocity component v. We will in our example take account of effects due to one particular type of collisions between ions and neutral atoms, namely collisions in which the ion and the atom are exchanging electric charge. That is, a fast ion and a slow atom before the collision are transformed into a fast neutral atom and a slow ion after the collision.

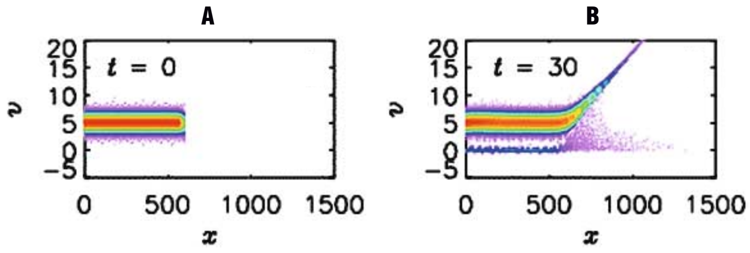

Figure 1. Phase-space snapshots of ion distribution function

Results from the simulation are plotted as phase-space plots in Figure 1a) and 1b), the former displaying the initial state at time $t = 0$, the latter the development at a later stage. In figure a) the plasma beam with a mean velocity of 5 velocity units has been allowed to penetrate a cold background neutral gas a distance of 600 length units. The distribution function f of the ions as a function of position x and velocity v are plotted using a color code, increasing values as the color changes from violet, through blue, green, yellow to red. The effects of the charge-exchange collisions are turned on at time $t = 0$.

Figure 1b illustrates several typical effects. First, due to the existence of unavoidable net charge densities near the beam head, and therefore the generation of localized electric fields in this region, ions near the beam head are accelerated to higher velocities and are therefore speeding away. This is seen as the upward pointing nose in the figure. Secondly, due to the effect of the charge-exchange collisions between the beam ions and the background neutral atoms, a population of slow ions are growing at the expense of the original ion beam. This is seen as the thick blue line at $v = 0$ in the figure and the more yellowish central part of the beam ion distribution function. This will, however, produce a double humped velocity distribution function for the total ion population. Under suitable conditions this situation will go unstable. Evidence of this is seen as a growing almost periodic spatial modulation of the double humped distribution function in the figure (ragged violet envelope of the blue line and increasing fuzziness of the low velocity part of the beam ion distribution function). In the end the phase space between the beam and the cold ion population will be filled with ions until the growth rate of the instability is saturated.

The result of this study is somewhat atypical in that it is a dissipative (collisional) process that is responsible for generating a free energy source (a double humped distribution function) that eventually leads to an instability and a turbulent state. We expect the results of the above and related examples to be of relevance for instance in chemical engineering, for the understanding of meteor dynamics and for the interaction of the solar wind with the interstellar medium.