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The Causality Principle: Complexity is the Limit

Synopsis: The Causality Principle has played an important role in the development of the theory of knowledge and constitutes a solid pillar in classical logic, used so often in daily life that we do not even realize it. Through causality, we may come to the conclusion that something happening regularly will always happen. This principle can successfully be applied to cases in which one has complete information on the situation involved. However, when complexity arises and uncertainties come into play, the principle becomes questionable. Under analysis, the presence of some randomness does not necessarily impede the validity of the principle. It is when uncertainties get out of control that the validity of the principle can no longer be ensured. The more complex the nature of the situation, the more information is required. The lack of reliable information in the face of complexity imposes the limit on the validity of the principle.

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Classical formulation

The world consists of events. All our personal experiences, historical features and scientific observations can be described as an orderly succession of events. An analysis of the nature of possible events shows that these can be of two types: those which can only be conceived and those which are truly real. Examples of the former are mathematical objects such as a triangle which are simply mental objects; the latter are all the objects and events belonging to the real world, such as a person or an experience. The Causality Principle states that all real events necessarily have a cause. The principle indicates the existence of a logical relationship between two events, the cause and the effect, and an order between them: the cause always precedes the effect.

An important property of the principle is that it entails predictability. Suppose that two causes give rise to two effects, respectively. It is easy to infer that if both causes are equal, the corresponding effects are also equal. Equal causes have equal effects and vice-versa. This fact implies the existence of a law of conservation under which the distinction is conserved. If two situations are distinct, they will remain distinct in all further evolutions and have been distinct during all previous evolutions as well.

Deductive versus plausible reasoning

We have seen that causality tacitly assumes predictability, that is, the absence of randomness or of any factor that may disturb the systematic flow of events. A fully causal world would then be a world in which all events would be perfectly determined. However, an inspection of the real world leads to the conclusion that this is far from the case in actual fact. There are many situations, such as night following day, or the influence of gravity due to which all objects which rise will necessarily fall, in which this systematicity can be guaranteed but there are many others in which randomness wins the game. Deductive reasoning based on the application of the principle cannot always be the right way to proceed in order to reach knowledge since quite often one has to cope with complex situations in which complete information of the events involved is hardly possible. In such situations, one has to extract conclusions from sketchy knowledge. Plausible reasoning which substitutes certainties for probabilities that an event may occur is then the only way to proceed. Many of these cases can be found in daily life. The uncertainties in the evolution of the trade markets or in the weather forecast are situations we often face. Modern tendencies in science aimed at exploring the world of the small where changes are very rapid and unpredictable use probabilities instead of deterministic quantities. Examples in which probability is the principal object are commonly found in statistical physics, quantum theory and the theory of chaos. Nowadays, the importance of a probabilistic description of a system is recognized in all domains, not only in the classical experimental sciences such as physics, chemistry and biology, but in economics and psychology as well. In words of the famous physicist James Clerk Maxwell:

> "The actual science of logic is conversant at present only with things either certain, impossible, or entirely doubtful, none of which (fortunately) we have to reason on. Therefore the true logic for this world is the Calculus of Probabilities, which takes account of the magnitude of the probability which is, or ought to be, in a reasonable man's mind."

But does randomness always imply the failure of the principle?

Deterministic versus probabilistic laws

Causality establishes a univocal relationship between cause and effect which can be expressed mathematically as R(C)=E. Many laws in science are of this type in that if C occurs E will always be observed. We say that C univocally determines E and, for this reason, these laws are called deterministic laws.

The following example illustrates the nature of a deterministic law. Suppose that we dissolve a chemical product in water. If the product is introduced at the left hand side of the container, the particles will go to the right thus making the mixture homogeneous, following the tendency that systems have to evolve towards equilibrium. The evolution of the particles' concentration can be described by the flux J accounting for the number of particles per unit area of a cross section and unit time, travelling from left to right. Non-equilibrium thermodynamics [1] establishes the form of this flux:

$$J = -D(c_R - c_L)/L$$

where L is the size of the container. This equation expresses that a difference in the concentration of particles on the left (c_L) and on the right (c_R) gives rise to the appearance of a current of particles travelling from left to right, from high to low concentrations. The magnitude of the current depends on a parameter D, the so-called diffusion coefficient, whose value is determined from the behaviour of the particles. The law thus formulated is causal. The concentration difference is the cause and the flux of particles the effect.

But let us now suppose that in the previous situation we do not know the precise location of the particles and only have information as to the probability of finding a given particle at a given position and time. As before, the particles accumulated on the left hand side go to the right part but we do not know the precise trajectories they follow. What changes now is not the concentration of particles but the probability. We observe that with the passing of time, the probability of finding a particle on the left decreases while increasing on the right until they balance each other out when the system reaches equilibrium. Despite uncertainties in the precise location of particles, a law similar to the previous one holds:

$$J = -D(P_R - P_L)/L$$

where now the flux is a probability flux, and P_R and P_L the probabilities of finding a particle on the left and on the right, respectively. We see that a difference in probabilities causes a probability flux from the left to the right. Despite the probabilities involved, this law is also causal. The cause is now the difference in probabilities, whereas the effect is the appearance of a flux of particles. But this causality refers to a situation in which we have only a partial knowledge of the system and the situation is thus intrinsically different from the previous one. To distinguish them we call the latter case *weak causality*. By means of this analysis, we may conclude that causality does not necessarily imply predictability but regularity, even for a probabilistic behaviour [2].

Causality in the lack of information

Let us consider again the previous experiment but with a new twist: the presence of an added source of particles injected from the right side. If the number of these injected particles exceeds those present on the left side, the probability of finding a particle on the right will be larger than that on the left and the difference in probabilities may change its sign, thus yielding a reverse current, in accordance with the former law. When we know of the existence of this source, we can proceed as we did previously, describing the situation by means of a causal law. But if we do not have information about the existence of such a source, we might conclude that causality is not fulfilled since the law predicts a forward current whereas what we observe is just the opposite. This example clearly shows that in order for causality to hold, we must have complete information about the situation under analysis. In the study of complex systems, one usually performs a reduction in the number of quantities by eliminating those one considers to be superfluous in the description of the system. Models proposed in different branches of science, as a caricature of what one thinks is going on, share this characteristic. Reductionism constitutes a powerful tool in science. In many cases, it is easy to identify those quantities that play a minor role in the behaviour of the system, but in others, this cannot be performed without distorting the description of the system. Complexity, inherent to these situations, then imposes a limit on the validity of the Causality Principle.

Conclusions

We can expect regular behaviour of a system when we have complete information about its behaviour. In this case, the Causality Principle holds. We have seen that the presence of randomness does not always imply violation of the principle. A small amount of randomness not reaching chaos does not necessarily impede the formulation of causal laws in terms of probabilities. This is a case encountered in many biological processes for which a thermodynamic description is possible [2]. We have called this *weak causality*. But when indeterminacy increases greatly, the system enters an unpredictable regime in which causality cannot be ensured.

References

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[2] J.M. Rubi, The long arm of the second law, *Scientific American*, November 2008, 62–67