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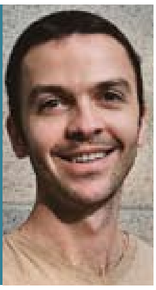
Objective Reality with Quantum Mechanics:

Can Spins Really Dance?

Introduction

Although our understanding of nature constantly improves and the fundamental concepts in science have been extended and refined, the opportunities in basic research are not becoming more scarce. On the contrary, there is always an abundance of fascinating problems. After some ‘stagnation’ of fundamental physics at the turn of the 19th century, when it seemed that all basic questions were being sorted out, mainly under the impression of the successes of the Lagrangian formulation of classical mechanics and the ground-breaking theories of statistical mechanics and classical electromagnetism, the 20th century ushered in the era of Einstein’s special and general relativity and the quantum-mechanical formulation of microscopic phenomena. The following discussion concerns an interplay of special relativity and quantum mechanics in electron transport in semiconducting structures, so-called spin-orbit interaction. Please note that a full merger of relativistic and quantum paradigms yet to be achieved. More specifically, the gravitational forces have not being incorporated into the ‘standard model’ of elementary particles and interactions. However, these issues are beyond the scope of this paper, and, in fact, outside the field of condensed matter physics as a whole.



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Spin

Let us start with a quick digression into discussing what spin is and the role it plays in physics. Spin, or the intrinsic angular momentum (which characterizes the amount of rotational motion within a particle), is a fundamental quantum-mechanical property of elementary particles. First of all, according to its magnitude, spin divides all elementary particles, as well as most known compound particles, such as atoms, and many effective quasi-particles in collective phenomena, into two categories: bosons and fermions. Bosons are particles that have integer value of spin in terms of the spin quantum $\hbar$, which is called Planck’s constant. Fermions are particles with half-integer value of spin. In other words, the former have spin 0, 1, 2, … while the latter $1/2$, $3/2$, $5/2$, … Planck’s constant $\hbar$ is one of the most celebrated fundamental constants in modern physics, appearing, e.g. in wave optics, quantum mechanics, statistical mechanics, and the knowledge of its value is being continuously improved. It equals approximately 1.05×10^{-34} J s, in the SI units. The main qualitative difference between bosons and fermions, besides their spin, is their quantum statistics. Quantum statistics, which has no analogs in the classical view of nature, describes what happens to the wave function (which reflects the state of quantum systems similarly to the way particle positions do for the classical systems) when two identical particles exchange positions. Although nothing physically observable should happen to the system when the positions of two indistinguishable particles are exchanged, the quantum formulation allows for an overall sign of the wave function to be flipped. It turns out the fermions do flip the sign, while the bosons do not. This may not seem like that much of a difference at first glance, which is true at high temperatures, but as the system starts to cool down, quantum statistics start having a profound impact on the macroscopic properties of matter. In particular, fermion statistics dictate that two identical particles cannot fall into the same quantum state, i.e. the Pauli exclusion principle. Boson statistics, on the other hand, do not suffer from this restriction, and particles tend to exploit this freedom at low temperatures to minimize the total energy. In fact, at sufficiently low temperatures, a macroscopic body consisting of identical bosons may undergo a phase transition into a different state of matter when a sizable portion of particles collapse into the same lowest-energy quantum state. This is known as Bose-Einstein condensation, resulting in a macroscopic object with pronounced quantum properties.

Figure 1: Rotating top: Its moment of rotation is closely analogous to the spin property of quantum-mechanical particles, especially for large spins.

Although it follows from this discussion that spin is intricately related to the formulation of quantum mechanics, this does have an obvious classical analog (as most quantum 'observables' do). It turns out, nevertheless, the large spins (on the scale of one $\hbar$) are very similar to the classical top sketched in Fig. 1. This analogy, however, has to be taken with caution for small spins, such as electron's spin $\hbar/2$. I will say a bit more about the relationship between the classical top and the spin in the next section.

Quantum mechanics

One can follow the development of most quantum mechanics theories by starting with the 'modern', i.e., Lagrangian formulation of the classical

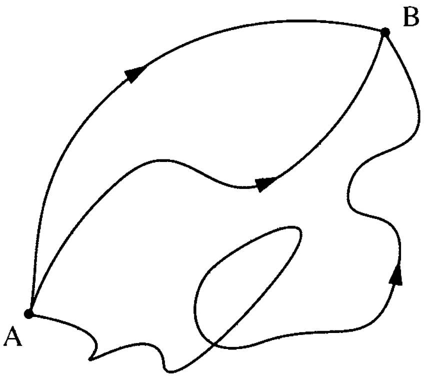

Figure 2: Quantum 'propagation' from A to B: All possible trajectories interfere, similar to wave interference in optics. There is always a path along which this interference is mostly constructive, which corresponds to the classical path. At high energies, almost all quantum weight lies close to this 'classical' path, which is only slightly 'fuzzy', as required by the Heisenberg's uncertainty principle.

mechanics (which dates back to the 18th century), based on the principle of the least action and defining the quantum-mechanical propagation of particles in terms of path integrals. This approach, which lays foundation to a very useful view of quantum mechanics, is ascribable to Richard Feynman [1]. A cartoon of path-integral formulation is shown in Fig. 2: While the classical propagation between two points *A* and *B*, separated in space and time, is given by certain deterministic trajectories, the quantum propagation is determined by an interference of all possible trajectories, which do not have to satisfy classical laws of motion.

It is also possible to cast the spin property of elementary particles in terms of the path-integral formulation for the classical top mentioned in the previous section. This is quite awkward, however, since one has to map from three-dimensional classical rotations to the quantum-mechanical representation of spin rotations, which gets mathematically involved and is rarely pursued in practice. Instead, one is completely comfortable thinking of spins as purely quantum-mechanical objects, without worrying too much about the exact classical-to-quantum mapping. As already mentioned, the difference between the quantum and classical spins diminishes for larger spins. That is why the classical rotating top can be very accurately described by classical mechanics. Even small spins, however, like those of elementary particles, still preserve some of the similarity with their classical analogs, so that the classical point of view can often be useful for qualitative purposes.

Spin-orbit interaction

One of the most fascinating properties of spin dynamics is the so-called spin-orbit interaction. It stems from the fact that spins, in addition to mechanical rotation, also carry a 'magnetic moment'. Loosely speaking, because the mechanical rotation is performed by the internal charge of the particle, we can envisage it as a tiny loop with circulating charge, essentially a solenoid magnet. This little magnet responds to the magnetic field, much like classical magnetized particles do: They precess and tend to align with the field as a compass arrow. To introduce the spin-orbit interaction, we only have to add one last ingredient: Einstein's special relativity.

Special relativity comes into play when particles move at speeds which are at least somewhat respectable as compared with the speed of light. Everyday phenomena around us are, however, too slow to reflect their fundamentally relativistic nature. That is why, not surprisingly, the Newtonian mechanics based on the Galilean theory of relativity (which is only approximately correct) has survived without challenge for centuries. It is natural that the theory that subjected Galilean relativity to doubt at the close of the 19th century involved Maxwell's equations of electromagnetism: The theory that describes the propagation of light. Classical electromagnetism subsequently formed the foundation for the special theory of relativity. According to the latter, all reference frames moving at a constant velocity in relation to each other are completely identical. At the same time, going from one frame to another can transform electricity into magnetism and vice versa, making the two phenomena fundamentally intertwined.

Spin-orbit interaction is one of the most illuminating consequences of this principle: In the frame of reference of a moving particle, an electric field is transformed into a mixture of electric and magnetic components. This means that a particle moving in the presence of an electric field will acquire a torque on its spin, corresponding to the spin precession in its rest frame magnetic field. Particle motion (also called orbital motion) and its spin dynamics are thus entangled. This turns out to be very important for electronic structures and transport in real materials, especially semiconductors. Although electrons do not diffuse too fast on average, they do undergo intermittent accelerations upon approaching positively charged atoms of the crystal lattice. This adds up to appreciable relativistic corrections to electron motion, reflected in the spin-orbit coupling. In summary, as an electron steps through the crystal lattice, it undergoes an intricate "dance" where its spinning motion is correlated with its trajectory through the lattice. Figure 3 shows a schematic cartoon of this phenomenon.

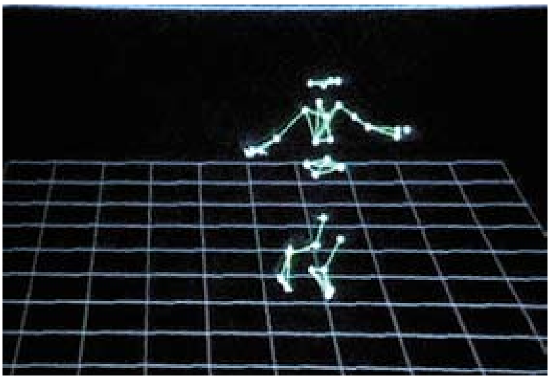

Figure 3: Spin-orbit 'dance': An electron's spinning is correlated with its motion through the atomic lattice. The phenomenon is possible due to an interplay of quantum mechanics and Einstein's special relativity.

The spin-orbit interaction has recently attracted considerable interest in the field of spintronics (the area which tries to utilize electrons' spin for novel nanotechnological devices). This stems in part from the possibilities to control spins with more easily accessible electric rather than magnetic fields. In particular, spin-orbit coupling can lead to the so-called 'spin Hall' effect, where a steady electron drift generated by an electric field induces a net transverse spin flow perpendicular to the drift direction. To understand this, imagine two teams of athletes competing in a marathon, distinguished by blue ('spin-up') and red ('spin-down') shirts. Suppose the blue shirts are instructed to pass slow runners on the left while the red shirts do it on the right. This will result in the net spin flow perpendicular to the marathon direction, since the up spins tend to 'flow' to the left and the down spins to the 'right'. In the spin Hall effect, the selectivity of the 'skewed scattering' which is analogous to the blue shirts bending to the left and the red shirts to the right, stems from the spinorbit interaction. There are other, more anomalous contributions to the spin Hall effect, but the essence is not too dissimilar from the spin-dependent skew scattering. This phenomenon has recently ignited a lot of interest in the spintronic community as a mechanism for efficient spin injection. It turns out, however, that when the spins are drawn out at the sides of the Hall bar (i.e., forced to exit the marathon in our analogy), they encounter spin-dependent selectivity at the Hall bar boundaries that impedes the net spin injection [3]. In our marathon picture: The blue shirts which are piled up to the left are prohibited from exiting on the left, while the red shirts cannot exit on the right. We found, however, that this selectivity can be suppressed by incorporating disorder into the edges, making side contacts more indifferent to the spins: The 'spectators' standing on the sidelines interfere with 'officials' trying to enforce the spin selectivity, restoring the spin injection.

During the program at the CAS, my colleagues and I have also worked on the problem of the interplay of spin-orbit interaction and quantum interference in ring-like structures [2]. We investigated spin-dependent phenomena which are governed by the interference, such as that sketched in Fig. 2, due to the trajectories propagating clockwise and counterclockwise along the ring. Unlike the spin-dependent skew scattering in the spin Hall effect, which is essentially a 'semiclassical' phenomenon, spin-dependent interference in rings is a purely quantum effect which reflects the wave-like nature of electrons. In both cases, however, the spin-orbit coupling plays a central role, leading to an amusing interplay between particle motion and its spin precession.

References

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