Parallel Thinking*

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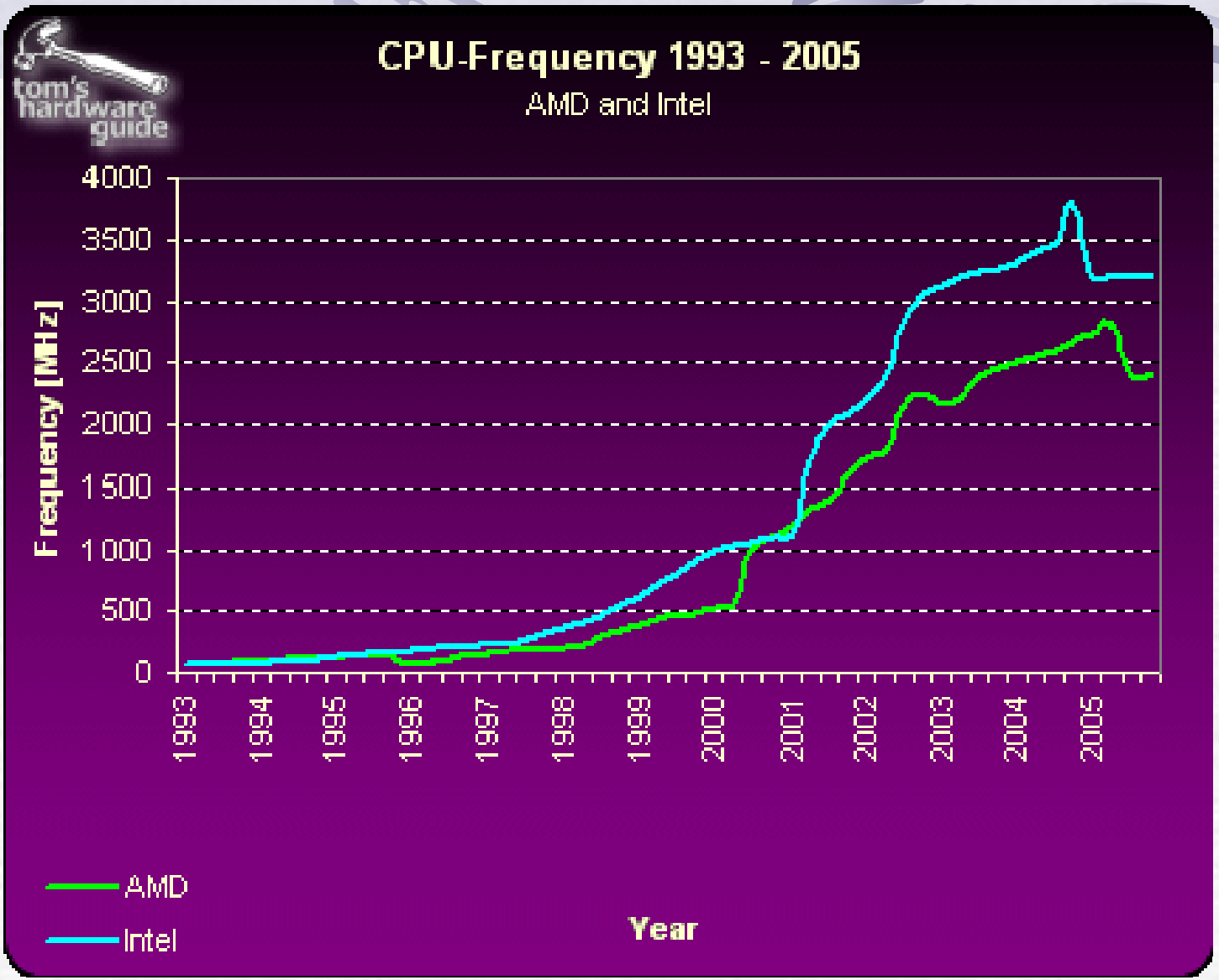
*PROBE as part of the Center for Computational Thinking



http://www.tomshardware.com/2005/11/21/the_mother_of_all_cpu_charts_2005/

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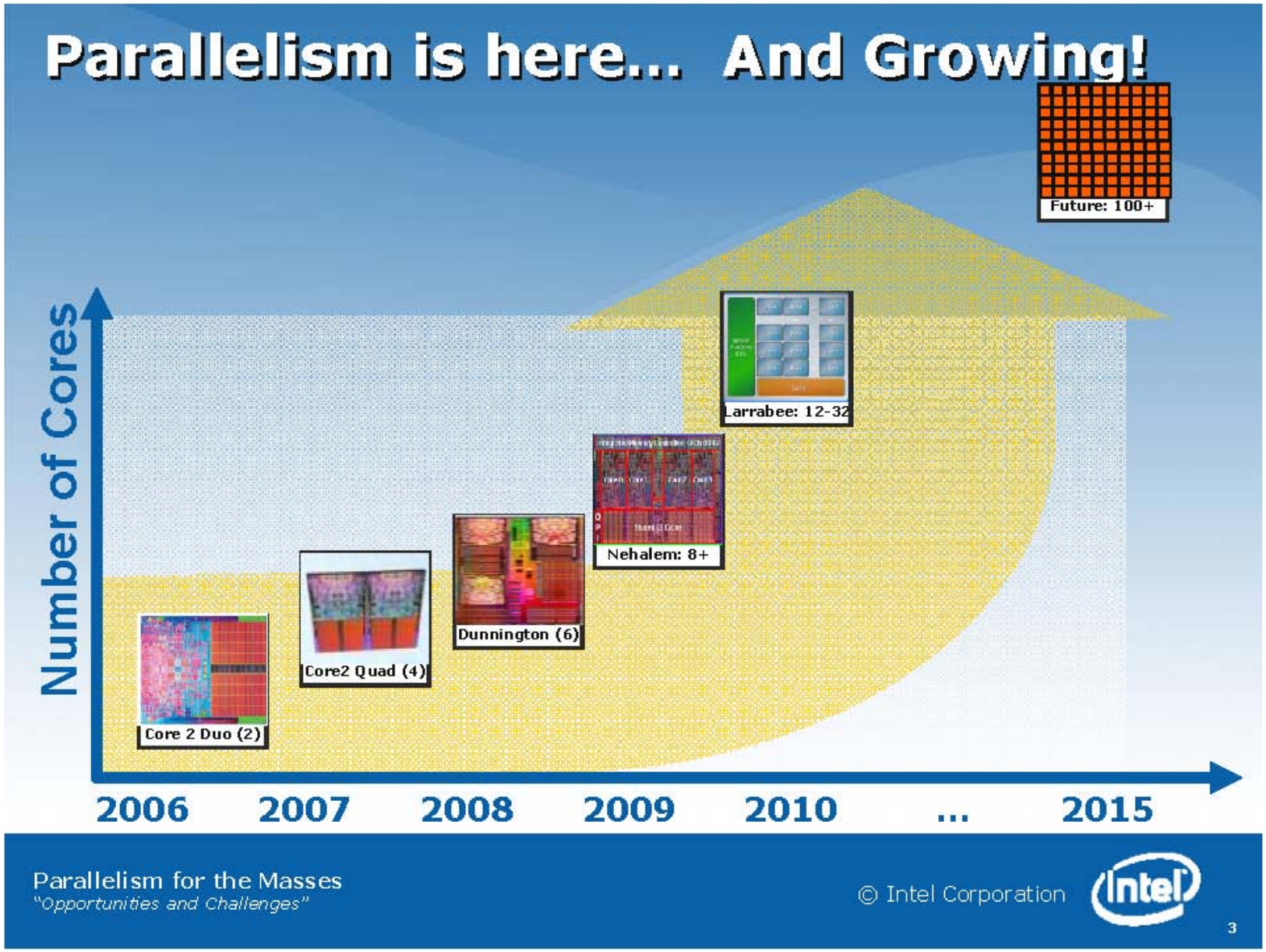


Andrew Chien, 2008

Parallel Thinking

How to deal with parallelism/concurrency:

Option I : Minimize what users have to learn about parallelism. Hide parallelism in libraries which are programmed by a few experts

Option II : Teach parallelism as an advanced subjet after and based on standard material on sequential computing.

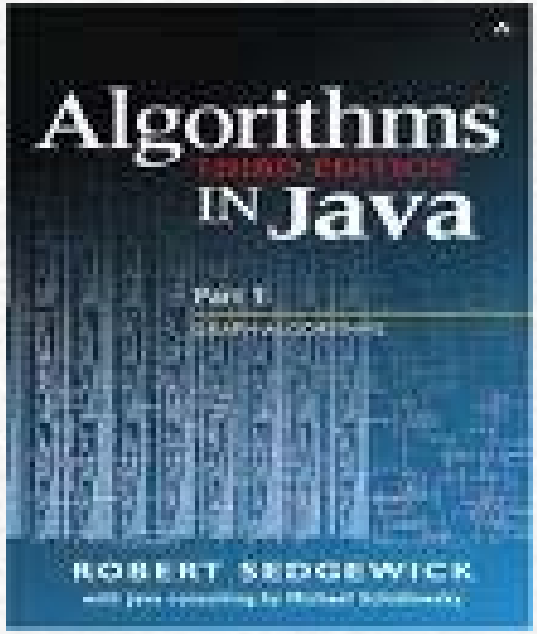
Option III : Teach parallelism from the start with sequential computing as a special case.

Parallel Thinking

- Could it be that it is more natural to think about parallel algorithms than sequential algorithms?
- If done right could parallel programming be easier than sequential programming, or at least for most uses?
- Are we currently brainwashing students to think sequentially?
- **What are the core parallel ideas that all computer scientists should know?**

Quicksort from Sedgewick

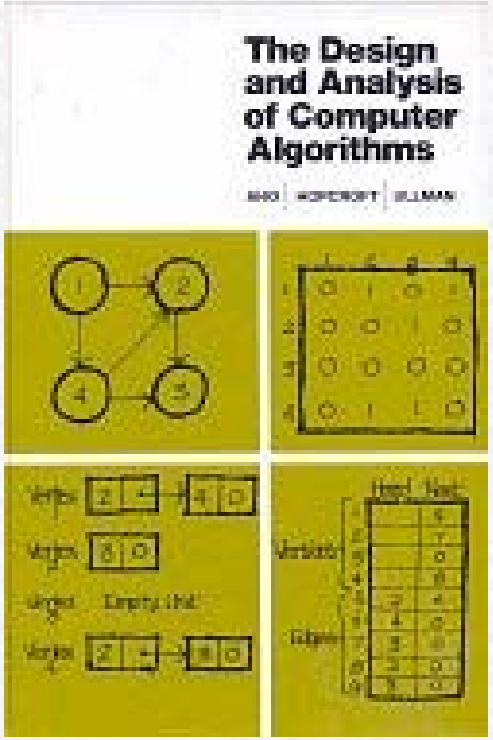
```
public void quickSort(int[] a, int left, int right) {
    int i = left-1;  int j = right;
    if (right <= left) return;
    while (true) {
      while (a[++i] < a[right]);
      while (a[right]<a[--j])
        if (j==left) break;
      if (i >= j) break;
      swap(a,i,j); }
    swap(a, i, right);
    quickSort(a, left, i - 1);
    quickSort(a, i+1, right); }
```



Sequential!

Quicksort from Aho-Hopcroft-Ullman

procedure QUICKSORT(**S**):
  if S contains at most one element **then return S**
  else
    begin
      choose an element **a** randomly from **S**;
      let $\mathbf{S_1}$, $\mathbf{S_2}$ and $\mathbf{S_3}$ be the sequences of
          elements in **S** less than, equal to,
          and greater than **a**, respectively;
      return (QUICKSORT($\mathbf{S_1}$) followed by $\mathbf{S_2}$
        followed by QUICKSORT($\mathbf{S_3}$))
end



Two forms of natural parallelism

Lesson 1

- Natural parallelism is often lost in “low-level” implementations.
  - We need “higher level” languages
  - We need to revert back to the core ideas of an algorithm

Quicksort in NESL

```
function quicksort(S) =
if (#S <= 1) then S
else let
  a = S[rand(#S)];
  S1 = {e in S | e < a};
  S2 = {e in S | e = a};
  S3 = {e in S | e > a};
  R = {quicksort(v) : v in [S1, S3]};
in R[0] ++ S2 ++ R[1];
```

Parallel selection

```
{e in S | e < a};
```

S = [2, 1, 4, 0, 3, 1, 5, 7]

F = S < 4 = [1, 1, 0, 1, 1, 1, 0, 0]

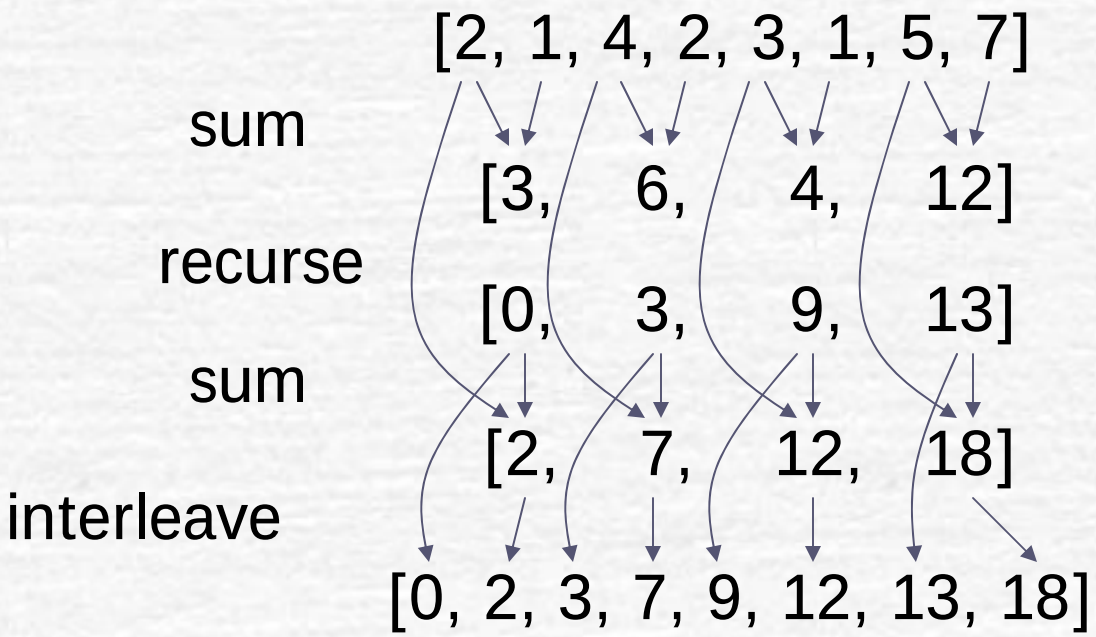
I = addscan(F) = [0, 1, 2, 2, 3, 4, 5, 5]

where F

R[I] = S = [2, 1, 0, 3, 1]

Each element gets sum of previous elements. Seems sequential?

Scan


[2, 1, 4, 2, 3, 1, 5, 7]
sum
[3, 6, 4, 12]
recurse
[0, 3, 9, 13]
sum
[2, 7, 12, 18]
interleave
[0, 2, 3, 7, 9, 12, 13, 18]

Scan code

```
function scan(A, op) =
if (#A <= 1) then [0]
else let
  sums = {op(A[2*i], A[2*i+1]) : i in [0:#a/2]};
  evens = scan(sums, op);
  odds = {op(evens[i], A[2*i]) : i in [0:#a/2]};
in interleave(evens,odds);,
```

A = [2, 1, 4, 2, 3, 1, 5, 7]

sums = [3, 6, 4, 12]

evens = [0, 3, 9, 13] (result of recursion)

odd = [2, 7, 12, 18]

result = [0, 2, 3, 7, 9, 12, 13, 18]

Lessons 2 and 3

- Just because it seems sequential does not mean it is

And

- When in doubt recurse on a smaller problem

Quicksort in Cilk++

```
seq quicksort(seq S) {
  if (S.length < 2) return S;
  double a = S[rand(S.length)];
  seq S1,S2,S3;
  cilk_spawn S1 = quicksort(lessThan(S,a));
  cilk_spawn S2 = eqTo(S,a);
  S3 = quicksort(grThan(S,a));
  cilk_sync;
  return S1.append(S2.append(S3));
}
```

Quicksort in Cilk++

```
int cnt=0;

seq quicksort(seq S) {
  cnt++;
  if (S.length < 2) return S;
  double a = S[rand(S.length)];
  seq S1,S2,S3;
  cilk_spawn S1 = quicksort(lessThan(S,a));
  cilk_spawn S2 = eqTo(S,a);
  S3 = quicksort(grThan(S,a));
  cilk_sync;
  S1.append(S2.append(S3));
}
```

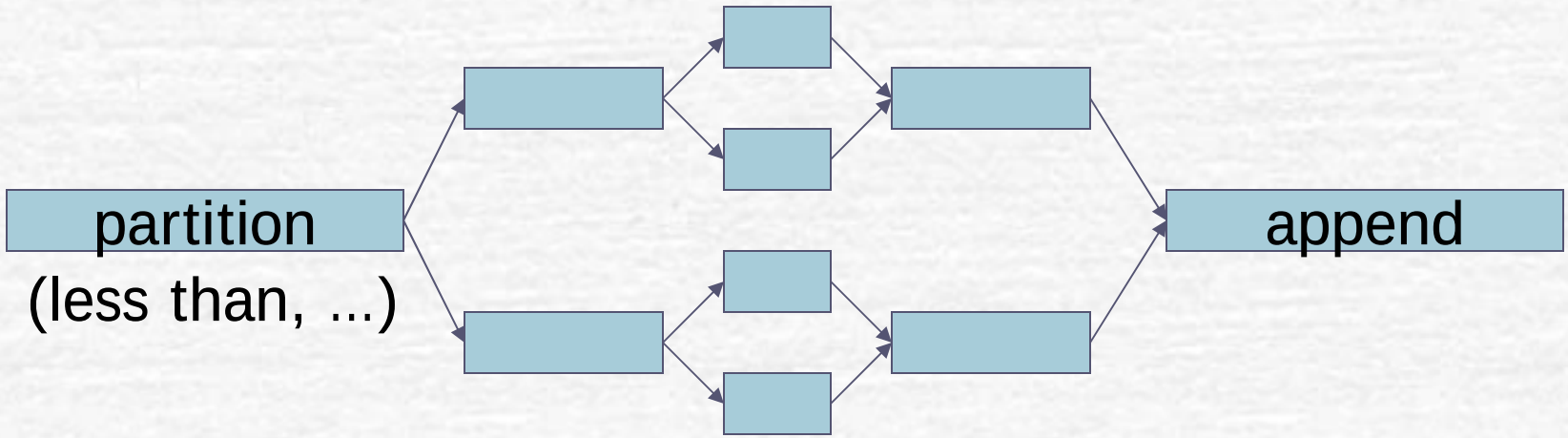
Lesson 4

- Deterministic parallelism is important
  - Functional languages
  - Race detectors
  - Using non-functional languages in a functional style

Atomic regions and transactions don't solve this problem.

Qsort Complexity

Sequential Partition
Parallel calls



Span = O(n)

Work = O(n log n)

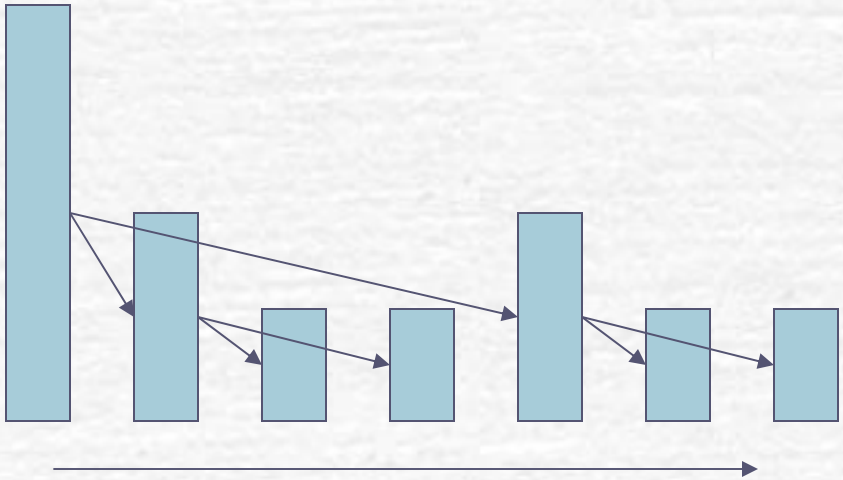
Not a very good parallel algorithm

Quicksort in HPF

```
subroutine quicksort(a,n)
integer n,nless,less(n),greater(n),a(n)

if (n < 2) return

pivot = a(1)
nless = count(a < pivot)
less = pack(a, a < pivot)
greater = pack(a, a >= pivot)

call quicksort(less, nless)
a(1:nless) = less

call quicksort(greater, n-nless)
a(nless+1:n) = less

end subroutine
```

Qsort Complexity

Parallel partition
Sequential calls



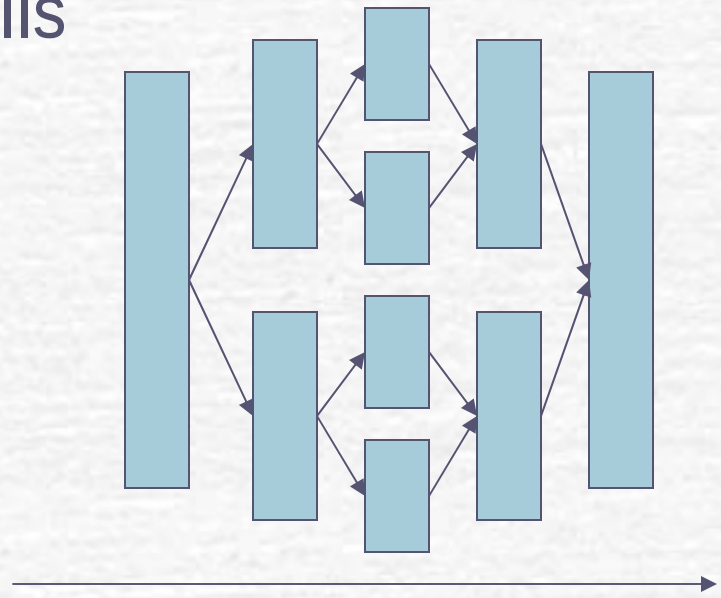
Span = O(n)

Work = O(n log n)

Still not a very good parallel algorithm

Qsort Complexity

Parallel partition
Parallel calls



Work = O(n log n)

Span = $O(\lg^2 n)$

A good parallel algorithm

Complexity in Nesl

Combining for parallel map:

```
pexp = {exp(e) : e in A}
```

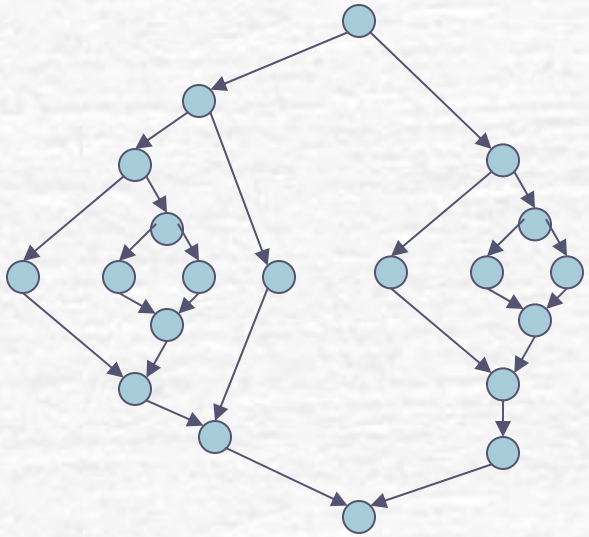
$$W_{\text{pexp}}(A) = \sum_{i=0}^{n-1} W_{\text{exp}}(A_i)$$ work

$$D_{\text{pexp}}(A) = \max_{i=0}^{n-1} D_{\text{exp}}(A_i)$$ span

Can prove runtime bounds for Various models:

e.g. $T = O(W/P + D \log P)$

Generally for a DAG



- Any “greedy” schedule for a DAG with span (depth) D and work (size) W will complete in:
  $T < W/P + D$
  time
- But, different schedules have very different space characteristics

Lessons 5, 6, 7 and 8

- Abstract cost models that are not machine based are important. Perhaps based in semantics of language.

and

- Work and span are reasonable measures

and

- Need to take advantage of both “data parallelism” and “function parallelism”

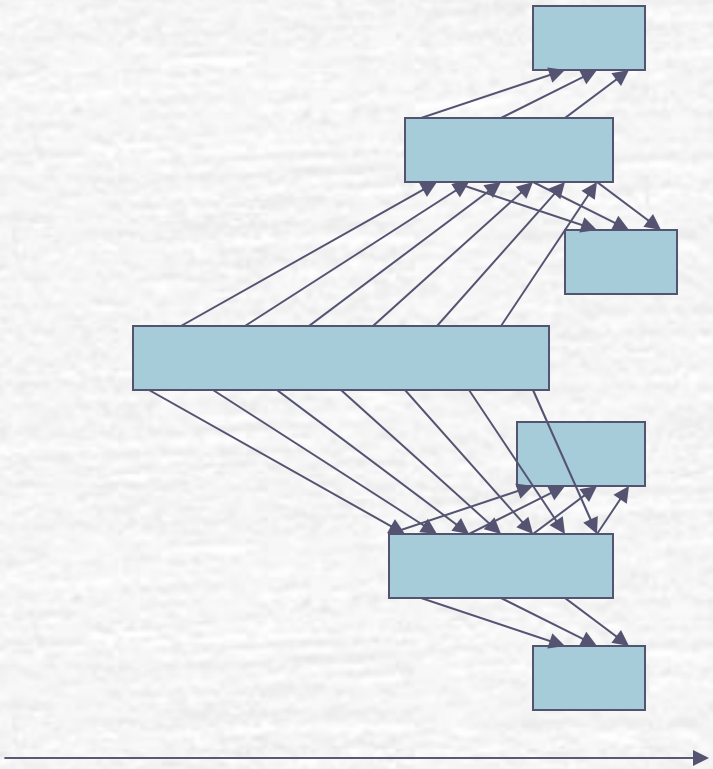
and

- Scheduling is important

Quicksort in Multilisp

```
(defun quicksort (L) (qs L nil))

(defun qs (L rest)
  (if (null L) rest
     (let ((a (car L))
           (L1 (filter (lambda (b) (< b a)) (cdr L)))
           (L3 (filter (lambda (b) (>= b a)) (cdr L))))
       (qs L1 (future (cons a (qs L3 rest)))))))

(defun filter (f L)
  (if (null L) nil
     (if (f (car L))
         (future (cons (car L) (filter f (cdr L))
         (filter f (cdr L))))
```

Quicksort in Multilisp (futures)



Work = O(n log n)

Not a very good parallel algorithm

Span = O(n)

Lesson 9

- Pipelining might be useful but one should analyze span before using it blindly

Example: Merging

```
Merge(nil,l2) = l2
Merge(l1,nil) = l1
Merge(h1::t1, h2::t2) =
    if (h1 < h2) h1::Merge(t1,h2::t2)
    else h2::Merge(h1::t1,t2)
```

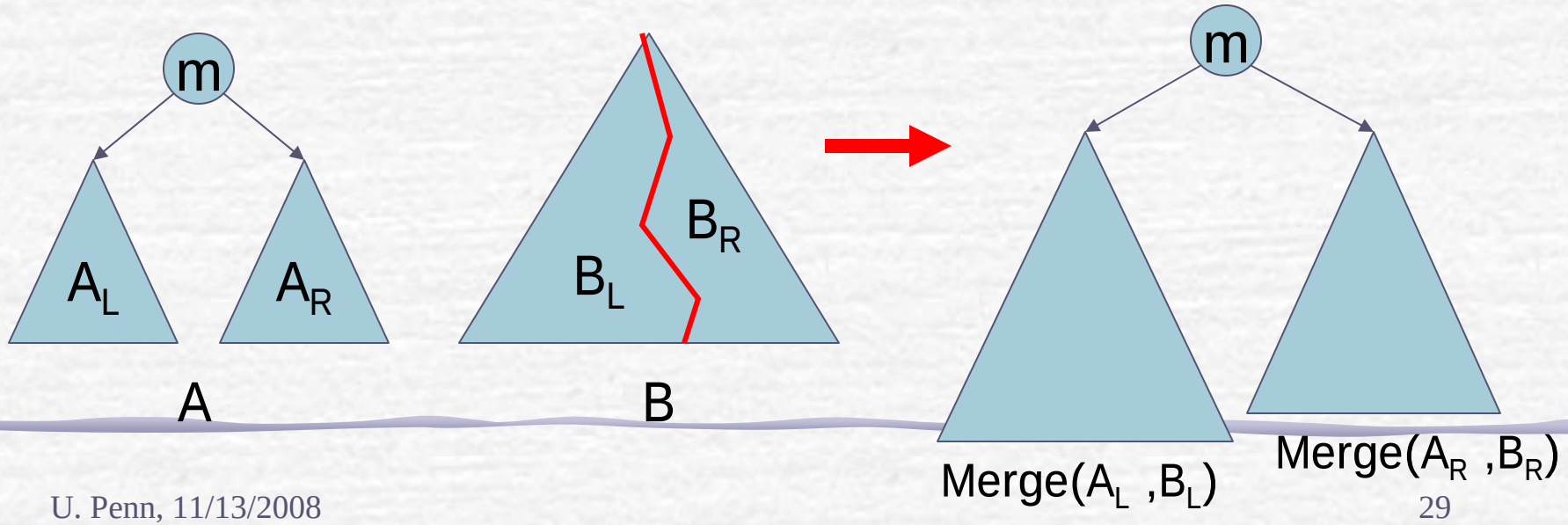
What about in parallel?

Merging

```
Merge(A,B) =
  let
    Node(A_L, m, A_R) = A
    (B_L ,B_R) = split(B, m)
  in
    Node(Merge(A_L,B_L), m, Merge(A_R,B_R))
```

Span = $O(\log^2 n)$
Work = $O(n)$

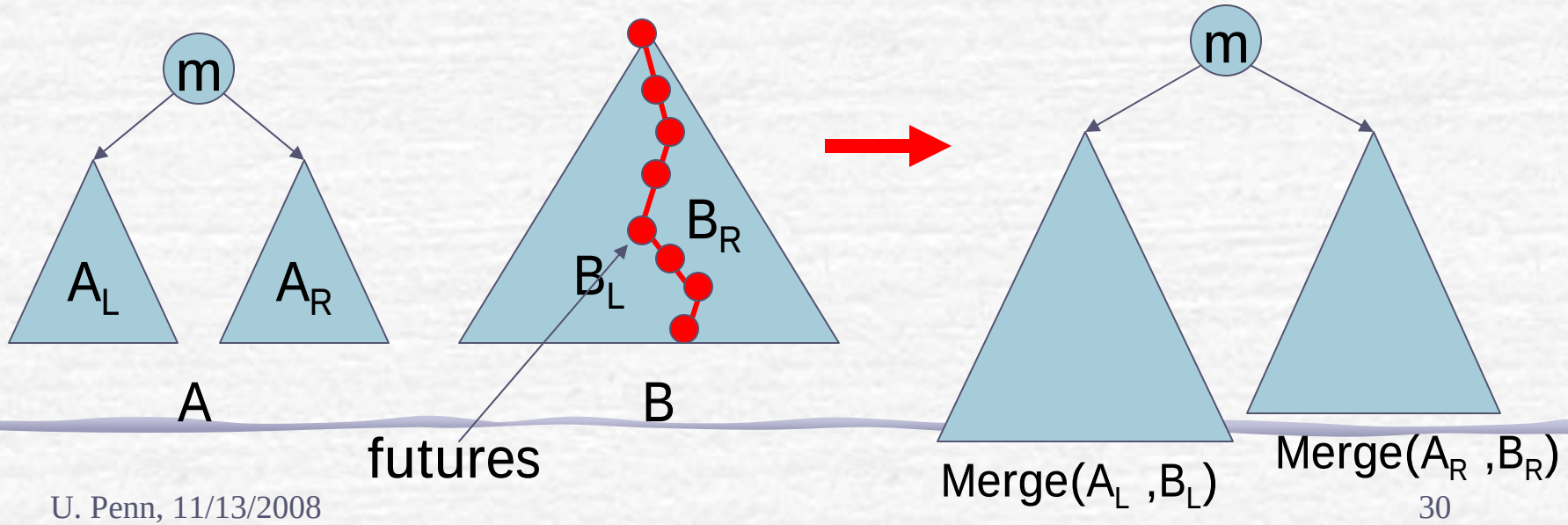
Merge in parallel



Merging with Futures

```
Merge(A,B) =
  let
    Node(A_L, m, A_R) = A
    (B_L ,B_R) = futureSplit(B, m)
  in
    Node(Merge(A_L,B_L), m, Merge(A_R,B_R))
```

Span = O(log n)
Work = O(n)



Lessons 10, 11, and 9.5

- Divide and conquer even more useful in parallel than sequentially

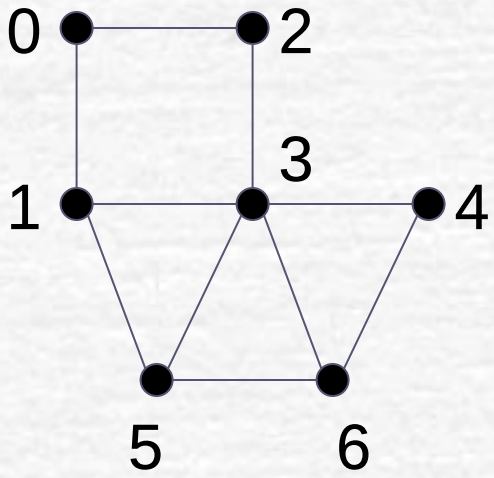
and

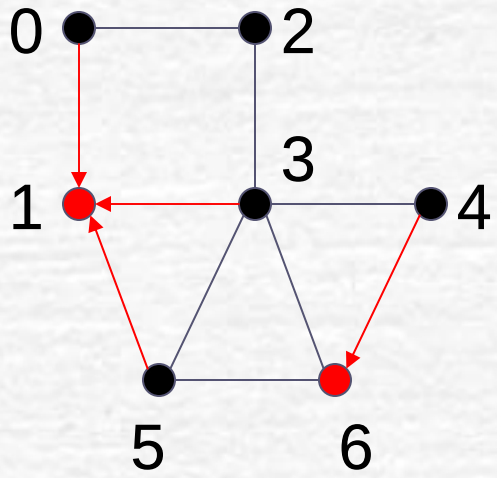
- Trees are better than lists for parallelism

and

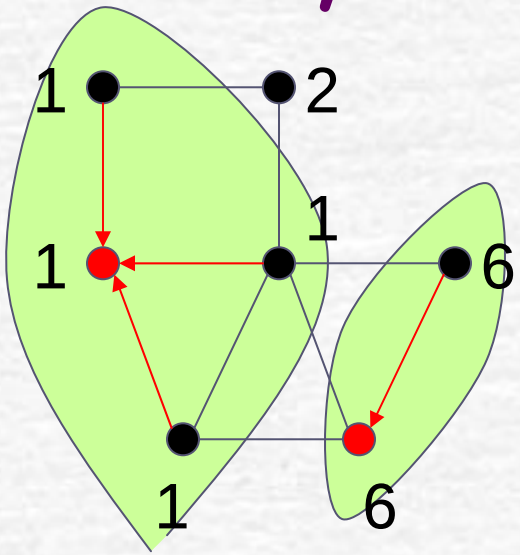
- (9.5) Pipelining is useful

Example : Graph Connectivity

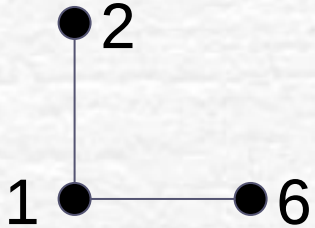




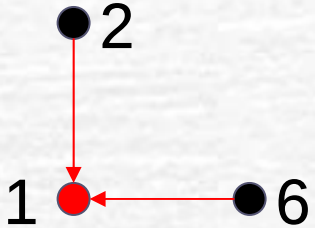
Form stars

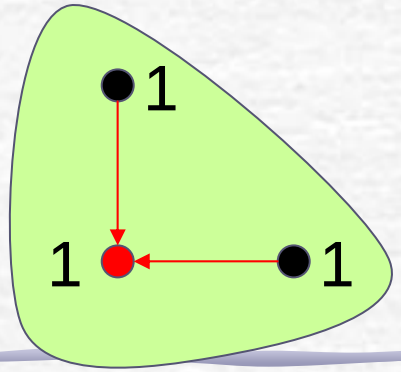


relabel

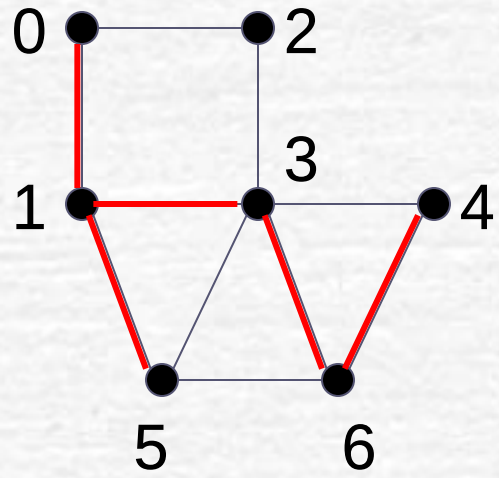


contract





Example : Graph Connectivity



Edge List Representation:

```
Edges = [(0,1), (0,2), (2,3), (3,4), (3,5),
  (3,6), (1,3), (1,5), (5,6), (4,6)]

Hooks = [(0,1), (1,3), (1,5), (3,6), (4,6)]
```

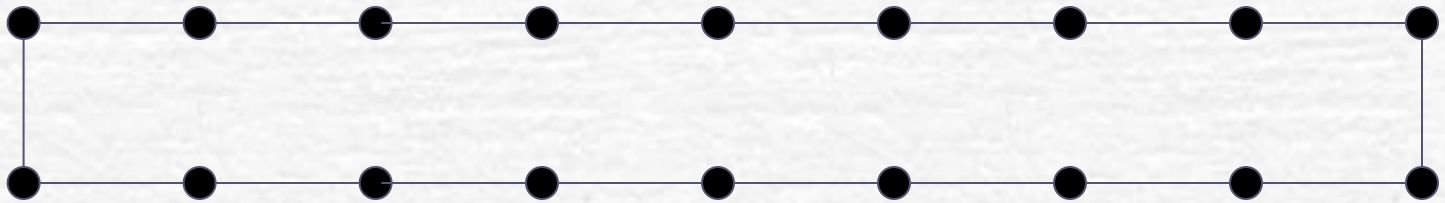
Example : Graph Connectivity

L = Vertex Labels, H = hooks   E = Edge List

```
L = L <- H;  /* Write tails to heads */

E = {(L[u],L[v])   /* update edge labels */
     : (u,v) in E
     | L[u]\=L[v]}; /* remove self edges */
```

Example : Graph Connectivity

- How do we find the stars?



The world looks the same from every node.
We need to break symmetry.

Example : Random Mate

L = Vertex Labels, E = Edge List

```
function connectivity(L, E) =
if #E = 0 then L
else let
  FL = {coinToss(.5) : x in [0:#L]};
  H = {(u,v) in E | Fl[u] and not(Fl[v])};
  L = L <- H;
  E = {(L[u],L[v]): (u,v) in E | L[u]\=L[v]};
in connectivity(L,E);
```

D = O(log n)
W = O(m log n)

Lessons 3, 12, 13

- When in doubt recurse on a smaller problem

and

- We often need to break symmetry

and

- Sampling is useful

The Lessons

General:

1. Natural parallelism is often lost in “low-level” implementations.
2. Just because it seems sequential does not mean it is

Model and Language:

4. Deterministic parallelism is important
5. Abstract cost models that are not machine based are important.
6. Work and span are reasonable measures
7. Need to take advantage of both “data” and “function” parallelism
8. Scheduling is important

Algorithmic Techniques

3. When in doubt recurse on a smaller problem
9. Pipelining is useful, with care
10. Divide and conquer even more useful in parallel
11. Trees are better than lists for parallelism
12. We often need to break symmetry
13. Sampling is useful

What else

Non-deterministic parallelism:

- Races and race detection
- Sequential consistency, serializability, linearizability, atomic primitives, locking techniques, transactions
- Concurrency models, e.g. the pi-calculus
- Lock and wait free algorithms

Architectural issues

- Cache coherence, memory layout, latency hiding
- Network topology, latency vs. throughput
- ...

...

Acknowledgements

This talk has been based on 30 years of research on parallelism by 100s of people.

Many ideas from the PRAM (theory) community and PL community

Conclusions/Questions

Should we teach parallelism from day 1 and sequential computing as a special case?

Could teaching parallelism actually make some things easier?

Are there a reasonably small number of core ideas that every undergraduate needs to know? If so, what are they?