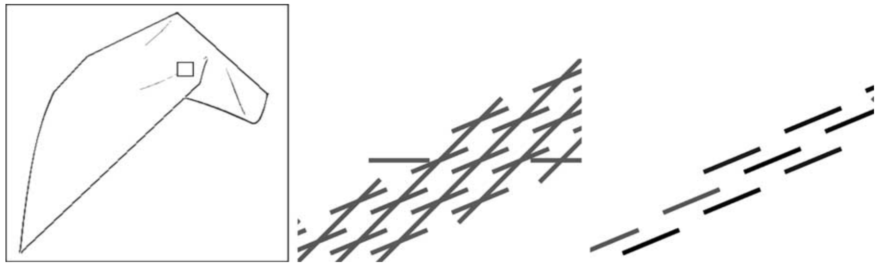

Fig. 10. Performance of our cooperative model for boundary and edge detection and localization. (Left) the sheet edge map and a region of interest. (Middle) Zoom on the initial edge measurements [23] in the region of interest. (Right) Result of 5 iterations of relaxation labeling [20] with Co-circularity compatibilities. Note the localization of the edge and the elimination of sparious measurements.

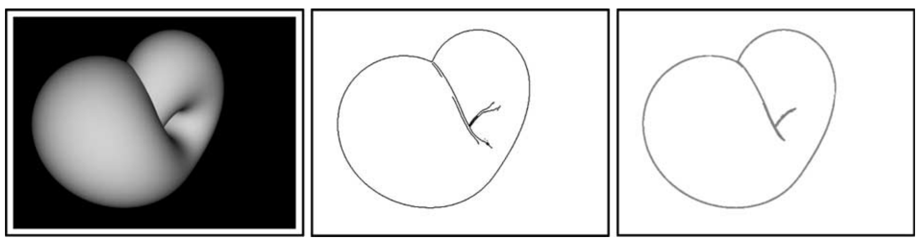

Fig. 11. The motivation from differential topology for early vision. (Left) The image of the Klein bottle shows how "T"-junctions can arise from occlusion relationships (e.g., at the top and center of the figure), and how certain interior edges can end (e.g., where the fold smoothly joins the body). (Middle) The Canny edge structure is inconsistent with both of these topological observations. Notice how the boundary "T"-junction is not connected, how it smooths the outline, and how the interior folds are blurred into the shading. (Right) Output of the logical/linear operator [23]. Notice how the "T"-junctions are maintained, and how the contours end at cusps.

## 4. Intra-columnar processes

The above example shows that boundaries need not be (in fact, are rarely) straight. Viewed locally, then, an approximation to a boundary over a very short distance is the tangent to the (boundary) curve. Viewed over a slightly larger neighbourhood, curvature begins to matter. Thus there are two problems that need to be addressed: (i) estimating tangents and (ii) estimating curvature.

Determining tangent directions with linear operators can be difficult, because different types of structure may fall within a single receptive field which would then be (inappropriately) averaged together. Thus we first consider the logical subunits of receptive fields and a mechanism to implement the non-linearities among them. The goal of these non-linearities is to prevent inappropriate types of structure from being combined into erroneous tangent estimates.

These local tangents are then combined to provide curvature responses, based on the idea that curvature can be viewed geometrically as "deviation from straightness". Measurements over different spatial scales are utilized in this case. The result is two local circuits that illustrate the elaboration of function across layers within a single orientation column. We now sketch each in turn, including new simulation results for the non-linearities in tangent estimation.

### *4.1. Building orientation response through logical subunits and shunting inhibition*

Boundaries that arise from surface folds, as discussed above, usually imply an intensity difference in the direction normal to the edge, with a generic dropoff in contrast in a neighbourhood around the edge (we shall consider this dropoff in detail in Section 6). Cracks appear as dark lines, with a bright-dark-bright contrast variation normally; while extended highlights can appear oppositely in contrast. Clearly, around "edges", there are different photometric events in in the direction normal to the edge [23].

In the tangential direction along an edge, continuity conditions arise. For the edge to be part of a boundary curve, locally its tangent must exist (almost everywhere) which implies that the limit as one point approaches another along it must exist. This can be realized as similar contrast signs along an edge element, for example.

Linear operators (or cells that combine receptive field subunits linearly) would respond similarly to various contrast situations. They would respond to both cracks and edges, and low-contrast edges and complicated distributions of intensities would all give intermediate responses. In effect, all of these different physical situations would be mapped into a single response, making the inverse problem of labeling the type of physical