This last system is known as Cartan's *connection equation* (O'Neill, 1966), and $w_{12}(\vec{\mathbf{V}})$ is called the *connection form*. Since $w_{12}(\vec{\mathbf{V}})$ is linear in $\vec{\mathbf{V}}$, it can be represented in terms of $\{\hat{\mathbf{E}}_T, \hat{\mathbf{E}}_N\}$:

$$w_{12}(\vec{\mathbf{V}}) = w_{12}(a\ \hat{\mathbf{E}}_T + b\ \hat{\mathbf{E}}_N) = a\ w_{12}(\hat{\mathbf{E}}_T) + b\ w_{12}(\hat{\mathbf{E}}_N)\ .$$

The relationship between nearby tangents is thus governed by two scalars at each point. We define them as follows,

$$\begin{aligned} \kappa_T &\triangleq w_{12}(\hat{\mathbf{E}}_T) \\ \kappa_N &\triangleq w_{12}(\hat{\mathbf{E}}_N), \end{aligned} \tag{2.3}$$

and interpret them as tangential (κ_T) and normal (κ_N) curvatures, since they represent a directional rate of change of orientation in the tangential and normal directions, respectively.

While the connection equation describes the local behavior of orientation for the general 2D case, it is equally useful for the 1D case of curves. Now, only $\nabla_{\hat{\mathbf{E}}_T}$ is relevant and equation 2.2 simplifies to

$$\begin{pmatrix} \nabla_{\hat{\mathbf{E}}_T} \hat{\mathbf{E}}_T \\ \nabla_{\hat{\mathbf{E}}_T} \hat{\mathbf{E}}_N \end{pmatrix} = \begin{bmatrix} 0 & w_{12}(\hat{\mathbf{E}}_T) \\ -w_{12}(\hat{\mathbf{E}}_T) & 0 \end{bmatrix} \begin{pmatrix} \hat{\mathbf{E}}_T \\ \hat{\mathbf{E}}_N \end{pmatrix}. \tag{2.4}$$

In its more familiar form, where T,N, and κ replace $\hat{\mathbf{E}}_T$,$\hat{\mathbf{E}}_N$, and κ_T, respectively, this is the classical Frenet equation (O'Neill, 1966) (primes denote derivatives by arc length):

$$\begin{pmatrix} T' \\ N' \end{pmatrix} = \begin{bmatrix} 0 & \kappa \\ -\kappa & 0 \end{bmatrix} \begin{pmatrix} T \\ N \end{pmatrix}. \tag{2.5}$$

2.2 Integration Models and Projection Patterns of Horizontal Connections. The geometrical analysis discussed above and illustrated in Figure 6 shows how the relationship between nearby tangents depends on the covariant derivative: for *curves*, the connection is dictated by one curvature; for *texture flows*, or oriented 2D patterns, two curvatures are required. By estimating these quantities at a given retinal point $\vec{\mathbf{q}}$, it is possible to approximate the underlying geometrical object, and thus a coherent distribution of tangents, around $\vec{\mathbf{q}}$. This, in turn, can be used to model the set of horizontal connections that are required to facilitate the response of a cell if its RF is embedded in a visual context that reflects good continuation. Naturally, to describe such a local approximation and to use it for building projection patterns, the appropriate domain of integration must be determined. However, since RF measurements provide only the tangent, possibly curvature (Dobbins, Zucker, & Cynader, 1987; Versavel, Orban, & Lagae, 1990), but not whether the stimulus pattern is a curve (1D) or a texture (2D), it is necessary to consider continuations for both.