



FIGURE 4. The matrix of 144 filters obtained by training on ZCA-whitened natural images. Each filter is a row of the matrix  $\mathbf{W}$ . The ICA basis functions on ZCA-whitened data are visually the same as the ICA filters.

The full ICA transform from the raw image was calculated as the product of the sphering (ZCA) matrix and the learnt matrix:  $\mathbf{W}_I = \mathbf{W}\mathbf{W}_Z$ . The basis function matrix,  $\mathbf{A}$ , was calculated as  $\mathbf{W}_I^{-1}$ . A PCA matrix,  $\mathbf{W}_P$ , was calculated from equation (7). The original (un-sphered) data were then transformed by all three decorrelating transforms, and for each the kurtosis of each of the 144 filters was calculated, according to the formula:

$$K_i = \frac{\langle (u_i - \langle u_i \rangle)^4 \rangle}{\langle u_i^2 - \langle u_i \rangle^2 \rangle^2} - 3 \quad (19)$$

Then the mean kurtosis for each filter type (ICA, PCA, ZCA) was calculated, averaging over all filters and input data. This quantity is used to quantify the sparseness of the filters, as will be explained in the Discussion.

## RESULTS

The filters and basis functions resulting from training on natural scenes are displayed in Figs 3 and 4. Figure 3

displays example filters and basis functions of each type. The PCA filters, Fig. 3(a), are spatially global and ordered in frequency. The ZCA filters and basis functions are spatially local and ordered in phase. The ICA filters, whether trained on the ZCA-whitened images, Fig. 3(c), or the original images, Fig. 3(d), are semi-local filters, most with a specific orientation preference. The basis functions, Fig. 3(e), calculated from the Fig. 3(d) ICA filters, are not local, and look like the edges that might occur in image patches of this size. Basis functions in the column Fig. 3(d) (as with PCA filters) are the same as the corresponding filters, since the matrix  $\mathbf{W}$  (as with  $\mathbf{W}_P$ ) is orthogonal. This is the ICA-matrix for ZCA-whitened images.

In order to show the full variety of ICA filters, Fig. 4 shows, with lower resolution, all 144 filters in the matrix  $\mathbf{W}$ . The general result is that ICA filters are localized and mostly oriented. Unlike the basis functions displayed in Olshausen & Field (1996), they do not cover a broad range of spatial frequencies. However, the appropriate comparison to make is between the ICA basis functions,