gamete. If $t[j] = 0$, then there exists some taxon $t'$ for which $t'[j] = 0$ and $t'[i] = 1$ and therefore $t'[i'] = 1$ and again $(0, 1)$ cannot be a newly introduced gamete. Finally consider any pair of characters $j, j'$. If taxon $t$ introduces gamete $(0, 1)$, then there exists some taxon $t'$ with $t'[j] = 0$ and $t'[i] = 1$ If $t'[j'] = 1$, then $(0, 1)$ cannot be a new gamete. If $t'[j'] = 0$, then $t[j'] = 0$ and not 1. The case when $(1, 0)$ is introduced by $t$ is symmetric. Finally if $t$ introduces $(1, 1)$ then consider any taxon $t'$ with $t'[i] = 1$. It has to be the case that $t'[j] = t'[j'] = 1$, and therefore $(1, 1)$ cannot be a newly introduced gamete. ■

We now have the following lemma.

*Lemma 4.5:* In every optimum phylogeny $T^*_M$, the conflict graph on the set of taxa in $T^*_M$ (Steiner vertices included) is the same as the conflict graph on $M$.

*Proof:* We say that a subgraph $F'$ of $F$ is the same as an edge labeled tree $T$ if $F'$ is a tree and $T$ can be obtained from $F'$ by suppressing degree-two vertices. A phylogeny $T$ is contained in a graph $F$ if there exists an edge-labeled subgraph $F'$ that is the same as the edge labeled (by function $\mu$) phylogeny $T$. We know from Lemma 4.3 that all optimum phylogenies $T^*_M$ for $M$ is contained in the (extended) Buneman graph of $M$. Lemma 4.4 shows that the conflict graph on $M''$ (and therefore on the extended Buneman graph of $M''$) is the same as the conflict graph of $M$. ■

*Lemma 4.6:* The probability that all guesses performed at Step 2c are correct is at least $2^{-q}$.

*Proof:* Implementation: We first show how to perform the guess efficiently. For every character $i$, we perform the following steps in order.

1) if all taxa in $M0$ contain the same state $s$ in $i$, then fix $r[i] = s$
2) if all taxa in $M1$ contain the same state $s$ in $i$, then fix $r[i] = s$
3) if $r[i]$ is unfixed then guess $r[i]$ uniformly at random from $\{0, 1\}$

Assuming that the guess at Step 2a (Figure 4) is correct, we know that there exists an optimum phylogeny $T^*_{M_j}$ on $M_j$ where $c(v)$ mutates exactly once. Let $e \in T^*_{M_j}$ s.t. $c(v) \in \mu(e)$. Let $r'$ be an end point of $e$ s.t. $r'[c(v)] = 1$ and $p'$ be the other end point. If the first two conditions hold with the same state $s$, then character $i$ does not mutate in $M_j$. In such a case, we know that $r'[i] = s$, since $T^*_{M_j}$ is optimal and the above method ensures that $r[i] = s$. Notice that if both conditions are satisfied simultaneously with different values of $s$ then $i$ and $c(v)$ share exactly two gametes in $M_j$ and therefore $i, c(v) \in \mu(e)$. Hence, $r'[i] = r[i]$. We now consider the remaining cases when exactly one of the above conditions hold. We show that if $r[i]$ is fixed