$\cup K$ is called the *vertex set* and its elements are called *vertices.* The *dimension* of a simplex with d vertices is $d-1$. The dimension of K is the maximum dimension among its simplices. A simplex τ is a *face* of a simplex γ iff $\tau \subseteq \gamma$, and iff $\tau \neq \gamma$ we say that τ is a *proper* face of γ. We say that K is *pure* if every simplex is a face of a simplex of highest dimension. Let S be a subset of K. We call the collection of all simplices in S together with all their faces, $Cl(S)$, the *closure* of S. The *star* of a simplex is the set of its superfaces, $St(\sigma) = \{\gamma : \sigma \subseteq \gamma\}$. The *link* of a simplex σ is the set of simplices in the closure of its star that do not intersect σ, namely, $Lk(\sigma) = Cl(St(\sigma)) - St(\sigma)$.

Let E be a mapping from the vertices of K to R^m. We let $|\sigma|$ denote the convex hull of the images of their vertices of σ under E, and let $|K| = \cup_{\sigma \in K}|\sigma|$. We say that $|K|$ is an embedding of K iff for all simplices σ and τ it holds that $|\sigma| \cap |\tau| = |\gamma|$ where γ is their maximum common face (which may be empty). We say that K is a d-manifold (with boundary) iff $|K|$ is a d-manifold (with boundary). If K is a manifold of dimension d then the link of every $(d-2)$-simplex is a cycle of edges and vertices (*i.e.*, a 1-manifold). If K is a manifold with boundary, then the link of every $(d-2)$-simplex is either a cycle or a path, *i.e.*, a 1-manifold with or without boundary (see Figure 1 (a)). We will make use of this fact in our representation described in section 5.

An *ordering*, $\vec{s^d}$, of a d-simplex, s^d, is a total ordering of its vertices. An *orientation*, $\overline{s^d}$, of a simplex, s^d, is a maximal set of orderings which are even permutations of each other[a]. Every ordering $\vec{s^d}$ on a simplex implies an orientation $\overline{s^d}$ on the simplex, and every d-simplex, $d > 0$, has two possible orientations. The orientation $\overline{s^d}$ of a simplex *induces* an orientation $\overline{s^{d-1}}$ on every $d-1$ subsimplex—*i.e.*, for all $\vec{s^{d-1}} \in \overline{s^{d-1}}$ there exists $\vec{s^d} \in \overline{s^d}$ such that $\vec{s^{d-1}}$ is a prefix of $\vec{s^d}$.

For our purposes, a *d-pseudomanifold* is a pure d-complex where every $(d-1)$-simplex is contained in at most two d-simplices and where the dual graph is connected. The vertices of the dual graph are the d-simplices and the edges are the $(d-1)$-simplices. A d-pseudomanifold is *orientable* if its d-simplices can be given orientations in such a way that when they meet at a $(d-1)$-simplex s, they induce opposite orientations on s. Every orientable d-pseudomanifold has two possible orientations, which can be specified by the orientation of one of its d-simplices. If K is a d-pseudomanifold then the link of every $(d-2)$-simplex is a collection of disjoint cycles and/or paths (see Figure 1 (b)).

In this paper we will use the term *simplicial mesh* to refer to a pseudomanifold abstract simplicial complex with a given orientation.

4. Interface

In this section we present the interface for simplicial meshes that our data structure implements. It is a simplified version of an interface described in Ref. [40].

[a]An *even permutation* is a permutation reached by an even number of swaps.