Wavelet Soft Threshold Application on Compressive Sensing in Wireless Sensor Networks of SHM

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Abstract. In the Wireless Sensor Network (WSN) for structural health monitoring (SHM), usually, we transmit the original acquisition data and mutual exchange the large amounts of data between nodes. It is a challenge for WSN accuracy and interoperability data communication in real time. Meanwhile, as compressive sensing (CS) is a novel framework in SHM based on WSN, we use CS for data compression to reduce network traffic and energy loss before transmission. In this work, we introduced the soft threshold wavelet method in the CS framework to reduce the noise generated by signals in the WSN of SHM. Through the experimental demonstration, we verify the performance of the application of this method for SHM signal reconstruction which could ensure the accuracy of the data.

1 Introduction

It is well known that compressive sensing (CS) [1,2] combines sampling and compression together to realize sampling below Nyquist rate. Besides, reconstructing a sparse signal with much fewer measurements than traditional means can be achieved by using CS. Nowadays, with the development of CS theory, it has been applied to WSN such as medical imaging, remote sensing [3], structural health monitoring (SHM) [4,5] and so on.

However, in the SHM based on WSN, signal with noise is not suitable for CS process. Thus the original data should be de-noising before the CS compression. Compared with other de-noising methods, the wavelet soft threshold method is simple but effective for one-dimensional signal de-noising process. In this paper, we choose the soft threshold method for wavelet threshold quantization process which has effect on CS compression and reconstruction.

The paper is structured as follows. In section 2 we provide the process of noise reduction in CS which also substantiates our scheme. In section 3 experimental verification steps are described. Section 4 concludes the paper.

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2 Noise reduction on CS in WSN

As we all know that CS theory includes three parts which are the signal sparse representation, the measurement matrix ensuring the data minimal information loss which should be satisfied the Restricted Isometry Property (RIP) and the reconstruction algorithm using the no-distortion observed value to reconstruct signals.

And for any N-dimension discrete-time signal $x \in R^N$, introducing the $N \times N$ orthonormal basis matrix *Ψ,* the signal $x \in R^N$ can be expressed as:

$$x = \sum_{i=1}^{N} \Psi_i \alpha_i \quad or \quad x = \Psi \alpha \quad . \tag{1}$$

In CS application，there are a variety of choices for the orthonormal basis Ψ such as the Fourier transform and the wavelet basis of the Wavelet transform. Haar wavelet basis is the first choice as the preferred orthogonal transform base in our experiments with higher sparsity. But the practical analysis found that both sparse effects didn't reach our paper goal. Most the minimum value of the sampling points is very close to zero. However, due to the sparseness of statistical is determined by the number of non-zero values in the sparse signals, it is necessary to make appropriate process on the signal after thinning. In addition, the noise signal has a great impact on the signal sparse representation, so that the de-noising process must be executed. The study found that the wavelet soft threshold process just handles the above two problems simultaneously to de-noise signal, retain the larger value and remove the minimum value. Moreover, the threshold algorithm has low computational complexity, so that it is an easy but very effective method. The following is a brief introduction for wavelet threshold process which also concludes our scheme.

2.1 Wavelet threshold noise reduction

After the signal sparse decomposition in CS, it is necessary to eliminate noise and process threshold value to get better sparsity, improve the data compression ratio and obtain precise reconstruction.

A mathematical model of one-dimensional signal contained noise is usually defined as follows (2):

$$s(n) = x(n) + \sigma * e(n) \qquad (n = 0,1,2,...N-1) \tag{2}$$

Where $x(n)$ donates the original signal, $s(n)$ represents a signal with noise, $e(n)$ is a noise signal, and σ means noise intensity. In the simplest case, amusing $e(n)$ is Gaussian white noise and $\sigma = 1$. The purpose of the wavelet threshold noise reduction is to try to suppress the noise signal $e(n)$ in order to reconstruct the original signal $x(n)$.

Wavelet threshold de-noising steps：As we all know that the signal de-noising process essentially inhibits the unwanted part of the signal and restore the useful part.

The main steps of wavelet threshold de-noising for one-dimensional signal are consists of noise signal decomposition, threshold quantization and signal reconstruction from it. Since there have been some wavelet decomposition and signal reconstruction mature algorithms, then how to select the threshold and threshold quantization approach became the core of the wavelet threshold method which directly determine the quality of the noise signal reduction in a considerable degree. So we only discussed the important parts just mentioned above here.

The threshold Value Selection: There are basically two types of obtained thresholds for the wavelet threshold method as shown on the table 1.

Table 1. Two kinds of thresholds selection methods

Threshold selection method	Classification
Based on the original signal	Donoho-Johnstone threshold Birge-Masart penalty function and Penaltythreshold
Nonlinear wavelet transform threshold selection	Minimaxi variance threshold sqtwolog unbiased risk estimation threshold (rigrsure) heursure

Through many experiments, in this paper we finally choose the 'rigrsure' method: using the Stein unbiased or likelihood estimation principle of adaptive threshold selection, given a threshold value T to get its likelihood estimation, and then minimizing the non-likelihood T. The specific algorithm is as shown in the table 2:

Table 2. Unbiased risk estimation threshold (rigrsure) algorithm

Algorithm 1 The rigrsure process

1. Taking the absolute value of each element in the signal arranged in ascending sort, and then obtaining the square of the respective elements to get a new signal sequence $f(k) = \left(sort\left(\left|S\right|\right)\right)^2$, $(k = 0,1,2...,N-1)$ where the 'sort' denotes sequence;
2. If we take the k-th element square root of the threshold $f(k)$ which is $T_k = \sqrt{f(k)}$, $(k = 0,1,...,N-1)$, then the risk generated by this threshold is $Rish(k) = \left[N - 2k + \sum_{j=1}^{k} f(j) + (N-k) f(N-k)\right] \Big/ N$;
3. In accordance with the resultant risk curve which denoted as $Rish(k)$, we take the '$k_{\min}$' as the corresponding value of its minimum risk point, therefore, the rigrsure threshold calculation formula is $T_k = \sqrt{f(k_{\min})}$.

The threshold quantization process method: After getting the threshold, the important step is to quantify it which has two kinds' process methods as shown in the table 3.

Table 3. The threshold quantization process method

Threshold process classification		Approach
Compulsory de-noising process		All high frequency coefficients of the wavelet decomposition structure is set to 0， and filter out all the high frequency part
The threshold value process	Soft threshold	Compared the signal absolute value with the threshold value, set the point value to 0 which is not greater than the threshold, and make the point value which is larger than the threshold with the difference between them.
	Hard threshold	Compared the signal absolute value with the threshold value, set the point value to 0 which is not greater than the threshold, and keep the original value when the point value is larger than the threshold.

Analyzing the table 3, we found that Compulsory de-noising process is simple and the signal de-noising is smooth but it is easy to lose a useful component in the signal, however the threshold value process has taken the low-frequency and high-frequency part of the signal into account, especially the soft threshold is the smoother way as well as the Hard threshold could retain more of the characteristics of real signal spikes.

In this paper, we choose soft threshold as the threshold quantization process method which mathematical model is described as follows (3). In order to facilitate the description, amusing that SORH=’s’ denotes the soft threshold.

$$SORH = 's' \ : \ \eta^{S}\left(w,T\right)=\begin{cases} sign\left(w\right)\left(\left|w\right|-T\right) & \left|w\right|\geq T \\ 0 & \left|w\right|<T \end{cases} \tag{3}$$

3 Simulation and performance evaluation

In this paper, we use MATLAB to achieve signal compression and reconstruction. To get the effective and real data of the experiments, we designed a data acquisition experimental system which sensor node is a common node without compression function. The whole size of the original data about 208M is used to simulate the CS processing. In the simulation, we take Gaussian random matrix as the measurement matrix and using OMP as the reconstruction algorithm. During the process of noise reduction, the threshold selection criteria (TPTR) choose ‘rigrsure’ method and the data length N is 1024.

3.1 The evaluation standards for CS applications in SHM

Compression ratio (CR): The compression ratio is one of the indicators to measure the degree of data compression whose definition is the compression ratio between the original signal data quantity and the compressed data amount written as the follows (4), where N_o, N_{co} denote the signal data quantity and compressed data amount.

$$CR = N_o / N_{co} \tag{4}$$

Reconstruction error ξ: Reconstruction error is on behalf of the similarity degree of the reconstructed signal and the original one. It is an import indicator to measure the effects of data decompression after refactoring which formula is as (5), where $\hat{x}$, x separately indicated the reconstructed signal and the original one.

$$\xi = \frac{\|\hat{x} - x\|_2}{\|x\|_2}$$

(5)

3.2 The soft threshold value method experiment in CS

Make M_stand=K*log(N/K), and control the number of measurements in M_stand-98<=M<=M_stand+98. From figure 1, we can see that under the situation TPTR 'rigrsure', taking the soft threshold method on the signal which length is N=1024, the sparisity generated is K=99 and CR is 90.33%.

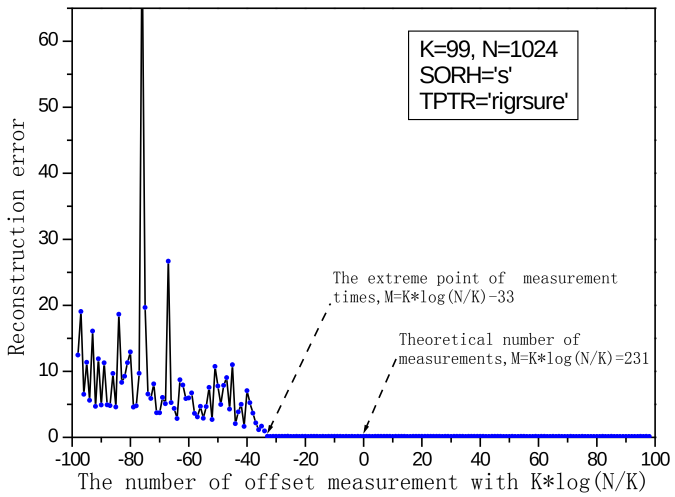

Fig.1. The soft threshold reconstruction error.

As we all know that the theoretical number of measurements is M_stand=231, actually, the number of measurements can be less than its theoretical times under the situation of that the signal length is larger. Figure 1 has indicated that the minimum number of measurements for soft threshold could not be less than M_stand-33, otherwise, reconstructing the signal will fail.

4 Conclusion

In this paper, the wavelet soft threshold quantization method was discussed. Through analyzing, we found that the threshold value process has taken the low-frequency and high-frequency part of the signal into account, especially the soft threshold is the smoother way. Experiments show that the RIC theory is also suitable for SHM based on WSN, which is proposed by Candes and Tao [6]. In SHM, we can draw more accurate conclusion $M \geq CK * \log(N/K) - 33$.

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