The New Facts about JPLRC for CAC

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Abstract. We had developed a kind of jumping piecewise linear requirement curve (JPLRC) before. It is a very general curve and can be utilized in connection admission control (CAC). This study extended our work and tried to realize the CAC. Specifically, some new facts are obtained to make the JPLRC used in CAC more possible and more efficient.

Keywords: Jumping piecewise linear requirement curve, Connection admission control, Quality of service.

1 Introduction

Future communication networks should guarantee Quality of Service (QoS) to users, and accordingly management control mechanisms are necessary for QoS provision. One of the important management control mechanisms is the connection admission control (CAC), which is the first gate of the network to provide QoS.

The CAC is associated with the scheduling algorithms of network systems. For providing QoS, Sariowan proposed a scheduling algorithm called SCED (Service Curve based Earliest Deadline first policy), which is a kind of Earliest Deadline First (EDF) algorithm [1]. Pyun also studied the SCED, where the service curve (SC) has a single rate and is leaky bucketed [2]. Hwang [3] studied the SCED for concave piecewise linear service curves (CPLSCs) and modified some equations of SCED in [1]. Recently, Hwang [4] developed a general curve, jumping piecewise linear requirement curve (JPLRC), which is useful in CAC. Previous works [1] and [2] suggested an algorithm in [5] to lower the complexity in CAC. Based on [4] and [5], this study obtained some new facts about the JPLRC and tried to facilitate the CAC.

The rest of this paper is organized as follows. The related background is given in Section 2. Section 3 revises the basic idea of [5] for JPLRC. The new important facts are illustrated in Section 4. Finally, conclusions are given in Section 5.

2 Background

From [4, (7) and Theorem 1], the unshifted JPLRC $R(t)$ can be written as

$$R(t) = \sum_{i=0}^{N} \rho_i : (\tau_i, \sigma_i)_{t-q} \tag{1}$$

where $\rho_i \geq 0$ for all i, $\rho_0 = 0$, $\rho_N = 0$ [4, Corollary 1], and $R(0_+) > R(0) = 0$. Furthermore, the availability curve $A(t)$ in [4, (16)] is rewritten below

$$A(t) = \sum_{j=0}^{M} \eta_j : (\alpha_j, \beta_j)_{t-q} \tag{2}$$

where $M + 1$ is the number of segments of $A(t)$, $\eta_0 = 0$, $\eta_1 > 0$, and η_i for $i > 1$ may be positive, zero, or negative. Notice that all the jumps of A must be negative jumps. [4]

The CAC algorithm is employed to find the minimum offset $\varphi_{\min}$ such that $R(t - \varphi_{\min}) \leq A(t)$. In [5], the basic idea is: "Only the convex points of availability curve A (F in [5]) and the concave points of arrival envelope E (A_f* in [5]) impose constraints on finding the minimum delay." By the basic idea, [5] developed a serial CAC algorithm for piecewise linear arrival envelopes, which finds two parameters in series and then takes their maximum to obtain the minimum delay provided by the network system. In this paper, we will generalize the basic idea for JPLRCs in section 3. Additionally, some new important facts are illustrated in section 4. The corresponding proofs are omitted for saving space.

3 The Basic Idea from [5] Revised for JPLRCs

Because the contact of R and A may be a line segment instead of a point, we modify the basic idea of [5] for JPLRCs. With the imaged flexion points [4, Definition 2], the revised basic idea is:

Theorem 1. Endpoints of a contact part of R and A must be a convex point of A or a concave point of R. The two endpoints become a single point for the case that the contact is a point rather than a line segment.

For the revised basic idea, two differences from [5] are: 1) the contact of R and A may be a line segment rather than a convex point of A or a concave point of E; and 2) the imaged line segments and flexion points defined above make the endpoints of a jump meaningful and accordingly conform to the basic idea.

4 New Important Facts

In this section, we present some new important facts that will be useful in developing the CAC algorithm. The important facts are about the key flexion point defined as:

Definition 1. A flexion point to decide the minimum offset $\varphi_{\min}$ is a key flexion point. The point on the other curve contacted by a key flexion point is called a key contact point.

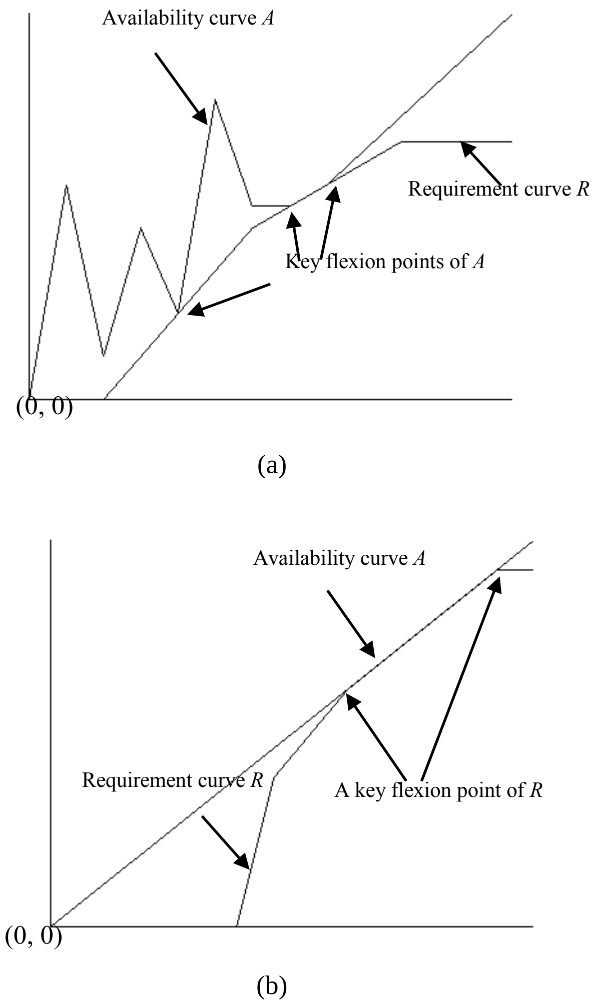

Fig. 1. The key flexion points: (a) for availability curve and (b) for requirement curve.

A key flexion point will penetrate through the other curve if RC moves further left. Specifically, if a key flexion point P is a convex point of A, further movement of R to the left will cause P below R. If a key flexion point P is a concave point of R, further movement of R to the left will cause P above A. Two examples of key flexion points are plotted in Fig. 1. The key flexion point is guaranteed to exist but may not be unique, and the minimum offsets found by those key flexion points are equal. A point P with coordinate (α, β) is denoted by $P(\alpha, \beta)$ and we say that α and β are the horizontal coordinate (HC) and the vertical coordinate (VC) of P, respectively. The important facts about the key flexion point and the key contact point are illustrated in the following Lemmas 1 and 2, which induce the most important fact that only partial curves rather than the whole curves of RC and AC are contactable. The contactable RC and AC are defined after Lemmas 1 and 2 respectively.

Lemma 1. For a key flexion point $P(\alpha, \beta)$ that is a convex point of A and the corresponding key contact point of R denoted by Q, the following properties hold:

a. $A'_{P+} > 0$;

b. All the convex points of A to the right of P are not lower than P;

c. Q is the terminal point of a horizontal segment H of R and is a real (not imaged) convex point of R if and only if $A'_{P-} \leq 0$ and Q is on H;

d. Q is the starting point of a horizontal segment H of R and is a concave point of R if and only if $A'_{P-} > 0$ and Q is on H;

e. Q is 1) the starting point of a horizontal segment of R that is a concave point; or 2) on a physical segment of R with positive slope.

For checking the convex points of A, Lemma 1e.1 implies that Q can be neglected, because Q is a concave point of R that will be checked while checking the concave points of R. Consequently, only the physical segments of R with positive slopes, excluding the starting points of horizontal segments, are necessary while checking the convex points of A. Therefore, the contactable RC and its inverse curve are defined as follows.

Definition 2. The contactable RC is defined as the collection of those segments of R with positive slopes in (1) and the contactable RC denoted by R_c can be expressed as

$$R_c(t) = \sum_{\{i:\rho_i > 0\}} \rho_i : (\tau_i, \sigma_i)_{t-q}. \tag{3}$$

Its inverse curve denoted by R_c^{-1} can be written as

$$R_c^{-1}(q) = \sum_{\{i:\rho_i > 0\}} 1/\rho_i : (\sigma_i, \tau_i)_{q-t}. \tag{4}$$

In the same way, the key flexion points of R and the contactable AC can be given as below.

Lemma 2. For a key flexion point P being a concave point of R and the corresponding key contact point of A denoted by $Q(\alpha, \beta)$, the following properties hold:

a. $A'_{Q-} > 0$;

b. Q cannot be located on a segment L of A with negative slope (including negative infinite);

c. If Q is on a horizontal segment L of A, then it is the starting point of L;

d. Q is always on a segment L of A with positive slope;

e. All the points of A to the right of Q are not lower than Q.

We call a convex point Q of A a convex point of v1-type, if $A'_{Q+} > 0$; and a convex point of v2-type, if $A'_{Q+} > 0$ and $A'_{Q-} \leq 0$. The points of v2-type is a subset of those of v1-type. While checking the concave points of R, Lemma 2d makes the resulting contactable AC consisting of some partial curves. Each partial curve of the contactable AC consists of some consecutive segments with positive slope. The starting point of the first segment of a partial curve must be a convex point of v2-type of A and can be excluded from the partial curve, because of Lemmas 2b and 2c. Accordingly, the domain of the first segment of the partial curve becomes left-open. The terminal point of the last segment of the partial curve must be a concave point Q with $A'_{Q+} \leq 0$. If $A'_{Q+} < 0$, Lemma 2b makes the last segment of this partial curve right-open. On the other hand, if $A'_{Q+} = 0$, Lemma 2c makes the last segment of a partial curve right-closed. For consistency, we let the domain of a line segment of a partial curve be a left-open right-closed (LORC) interval. Consequently, we can define the contactable AC and its inverse as follows.

Definition 3. Define the contactable AC A_c as the collection of those segments η_i: $(\alpha_i, \beta_i)_{t-q}$ of A in (2) with $\eta_i > 0$, where the original left-closed right-open segments are transferred into LORC segments. The contactable AC can be presented as

$$A_c(t) = \sum_{\{i:\eta_i>0\}}' [\eta_i : (\alpha_i, \beta_i)_{t-q}]_+ \tag{5}$$

where $[\eta_i$: $(\alpha_i, \beta_i)_{t-q}]_+$ denotes the domain-shifted segment of η_i: $(\alpha_i, \beta_i)_{t-q}$ and can be written as

$$q = \eta_i(t - \alpha_i) + \beta_i,\ \alpha_i < t \leq \alpha_{i+1},\ i = 1, \ldots, M,$$

and we let $\alpha_{M+1} = \infty$. Additionally, the inverse curve of A_c can be expressed as

$$A_c^{-1}(q) = \sum_{\{i:\eta_i>0\}}' [1/\eta_i : (\beta_i, \alpha_i)_{q-t}]_+ \ . \tag{6}$$

For the case that a convex point X of A with $A'_{X-} \leq 0$ and $A'_{X+} > 0$ contacts with the concave point Y of R with $R'_{Y-} > R'_{Y+} > 0$, both X and Y can be key flexion points. However, this case does not conform to Lemma 2a, but does conform to Lemma 1. Therefore, in this case, X is the key flexion point and Y is the key contact point. Lemma 1 integrated with Lemma 2 completely describe the facts about the key flexion point and the key contact point.

In fact, R_c of (3) is R of (1) with horizontal segments erased and is strictly increasing, so the following lemma hold:

Lemma 3. If x is in the domain of R_c, then $R_c(x) < R(x_+)$.

Because there are no positive jumps in A, $A_c(t)$ is an onto function with range $[0, \infty]$ and may be a multi-valued function. Therefore, $A_c^{-1}(q)$ is an inverse image (or called a set-valued function) as

$$A_c^{-1}(q) = \{t: A_c(t) = q\}.$$

We let max{ $A_c^{-1}(q)$} = max{t: $A_c(t) = q$} and have next lemma.

Lemma 4. Some properties of the A_c are as follows:

a. If a is in the domain of $A_c(t)$, then $A_c(a) > A(a_{-})$;

b. If a certain $q > A(a)$, then there exists $b > a$, b in the domain of A_c, to have $A(b) = A_c(b) = q$ and max{ $A_c^{-1}(q)$} > a;

c. For a jump at a, if $A_c(a) = q$, then max{ $A_c^{-1}(q)$} > a;

d. If $a \geq$ max{ $A_c^{-1}(q)$}, then $A(a) \geq q$; and If $A_c(a) \leq q$, then $a \leq$ max{ $A_c^{-1}(q)$}.

Because of domain-shift in A_c, if a (negative) jump is at a, then $A_c(a) > A(a)$. For $A_c(a) = q$, it is impossible to have max{ $A_c^{-1}(q)$} = a stated in Lemma 4c. Therefore, the point (a, $A_c(a)$) is included in A_c for consistency.

5 Conclusion

This work extended our previous work of JPLRC to make a CAC algorithm more possible. JPLRCs are very general curves and will be useful in CAC. Therefore, we study the properties of JPLRCs and find some new facts about the key flexion points and contactable curves. With these facts, we believe an efficient CAC algorithm can be developed. It is the future work and is ongoing now.

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