

# Averaged Channel Capacity for ADF Cooperative Systems with Burst Symbol Transmission over Quasi-Static INID Rayleigh Fading channels

Kyunbyoung Ko<sup>1</sup> and Sungmook Lim<sup>2</sup>

<sup>1</sup>Dept. IT Convergence and  
Dept. of Control and Instrumentation Engineering,  
Korea National University of Transportation,  
50 Daehak-ro, Chungju-si, Chungbuk, 380-702 Korea  
kbko@ut.ac.kr

<sup>2</sup>Dept. of Electronics Engineering,  
Korea National University of Transportation,  
50 Daehak-ro, Chungju-si, Chungbuk, 380-702 Korea  
(Corresponding author) smlim@ut.ac.kr

**Abstract.** In this paper, we propose the averaged channel capacity expression for adaptive decode-and-forward (ADF) relaying schemes based on burst M-ary symbol transmission over quasi-static Rayleigh fading channels. The derived analytical approach is verified based on the number of relays and data symbols and modulation orders. Its accuracy is confirmed by comparison with simulation results.

**Keywords:** channel capacity, ADF, burst transmission, Rayleigh fading channel

## 1 Introduction

The authors in [1] have shown the general approach applicable for both DF and ADF relaying and derived an exact bit error rate (BER) as well-known tractable forms. Note that even if it can give exact results [1], previous researches have been carried out under the assumption that relay nodes can detect symbol-by-symbol error. It is not practical and the performance based on this implies only an achievable lower bound. So far as we know, the practical approach covering burst transmission for ADF relay systems has not been addressed in the literature yet. Furthermore, no one has expressed the approximated outage probability expression as well-known tractable forms, which can explain how an erroneous detection at each relay affects both the received SNR and the average outage probability.

At first, we consider not symbol-by-symbol but burst-by-burst error detection for ADF relay systems. Based on this, we focus on the error-event at relay nodes for burst transmission and then, the probabilities of all possible error-events are derived as well-known forms. By considering burst transmission for ADF relay systems, we derive an averaged channel capacity expression over independent and non-identical

distributed (INID) Rayleigh fading channels, so that it can show a practical performance. Notice that the averaged channel capacity is approximated to the simplified expression for an arbitrary link SNR and modulation order. Numerical results obtained from analytical solutions and Monte-Carlo simulations are compared. Then, it is confirmed for ADF relay systems that the numerical result in [1] and the approximated performance in this paper can be the achievable bound and practical bound, respectively.

## 2 ADF Relay Systems with Burst M-ary Symbol Transmission

ADF relay systems have a source (S), a destination (D), and  $L$  relay (R)s. In this paper, it is assumed that S and  $L$  relays transmit over orthogonal time slot [1][4][5]. For the ADF relay systems, the received signals for the S-D, the  $r$ th S-R, the  $r$ th R-D links can be presented, respectively, as

$$\begin{aligned} y_0 [t] &= h_0 \sqrt{E_0} s [t] + n_0 [t] \\ y_{L+r} [t] &= h_{L+r} \sqrt{E_{L+r}} s [t] + n_{L+r} [t] \\ y_r [t] &= h_r \sqrt{E_r} \hat{s}_r [t] + n_r [t]. \end{aligned} \quad (1)$$

In ADF relay systems, the  $r$ th relay participates in transmitting the regenerated symbol of  $\hat{s}_r [t]$  only when burst messages are correctly decoded.

### 2.1 Error-Event of Relay Nodes with Burst Transmission

In order to derive the analytical method based on error-events at relays, let us define the  $p$ th error-event vector  $E^p$  as  $E^p = [e_1^p \cdots e_r^p \cdots e_L^p]$  and the total number of error-events is  $2^L$ . Generally, we can define that  $E^1$  is all-zero vector,  $E^{2^L}$  is all-one vector, and so on. Note that for the  $p$ th error-event,  $e_r^p = 0$  means the correct burst detection at the  $r$ th relay and  $\hat{s}_r [t] = s [t]$  for  $N_p < t \leq N_F$  with the probability of

$$P_{TM} (\bar{\gamma}_{L+r}) = \sum_{k=0}^{N_D} \binom{N_D}{k} (-1)^k E [P_S^k (\gamma_{L+r})] \quad (2)$$

and [2], [3]

$$E [P_S^k (\gamma_{L+r})] < \int_0^\infty a^k Q^k (\sqrt{b\gamma}) f_{\gamma_{L+r}} (\gamma) d\gamma. \quad (3)$$

Also,  $e_r^p = 1$  leads to  $\hat{s}_r [t] = 0$  with the probability of  $1 - P_{TM} (\bar{\gamma}_{L+r})$ . Furthermore, the probability of the  $p$ th error-event at DF relay systems is presented as [5], [6]

$$P r^p = \prod_{r=1}^L \left[ P_{TM}(\bar{\gamma}_{L+r}) \right]^{e_r^p} \left[ 1 - P_{TM}(\bar{\gamma}_{L+r}) \right]^{1 - e_r^p}. \quad (4)$$

We can obtain the approximated bound as

$$E \left[ P_S^k(\gamma_{L+r}) \right] \approx \frac{a_1^k}{1 + b_1 k \bar{\gamma}_{L+r}} = E \left[ P_S^k(\gamma_{L+r}) \right]_{App}. \quad (5)$$

Let us consider the case of combining signals from S-D and R-D links. Then, the combined instantaneous SNR can be written as

$$\gamma_C^p = \gamma_0 + \sum_{r=1}^L e_r^p \gamma_r = \sum_{r=0}^L e_r^p \gamma_r. \quad (6)$$

## 2.2 Closed-form Expression for Averaged Channel Capacity

The channel capacity, in the Shannon's sense, is an important performance metric since it provides the maximum achievable transmission rate under which the errors are recoverable. For the  $p$ th error-event, the average channel capacity can be approximated as

$$\bar{C}^p = \frac{BW}{(L+1) \ln(2)} \sum_{r=0}^L \pi_r^p e^{1/\bar{\gamma}_r} E_1(1/\bar{\gamma}_r) \quad (7)$$

where  $BW$  is the transmitted signal bandwidth [4]. Consequently, the averaged channel capacity can be shown as  $\bar{C} = \sum_{p=1}^{2^L} P r^p \bar{C}^p$ .

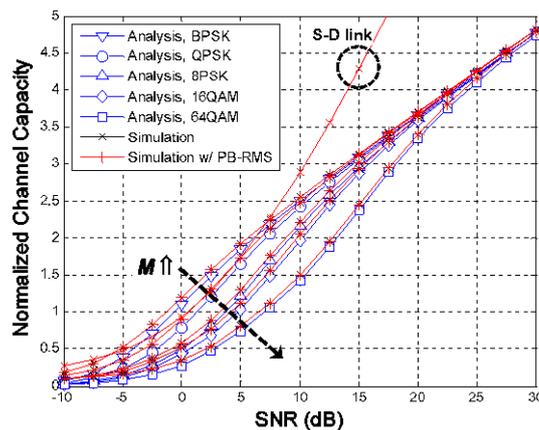
## 3 Numerical and Simulation Results

Fig. 1 shows the average channel capacity versus SNR for ADF relay systems. From this figure, we can find that the average channel capacity performance decreases in proportion to  $M$  (modulation order). The derived numerical results are well matched with simulation results. Consequently, it is confirmed that the derived analytical approach can be used as a general tool to verify effects of burst transmission on the average channel capacity and cooperative diversity gain over quasi-static Rayleigh fading channels.

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**Fig. 1.** Averaged channel capacity versus SNR (dB) with respect to different  $M$  ( $L=2$ ,  $N_D=32$ ,  $M=2, 4, 8, 16, 64$ ).