

## A Novel Weak Signal Detection Method with Unknown Frequency Uncertainty

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**Abstract.** Aiming at the difficult problem of weak signal detection, a weak signal detection method based on the compound system for weak signal with unknown frequency is proposed, which implements the detection of weak signal submerged in strong noise and provides for the basic theory and technical support for the protection of industrial processes to efficient and stable operation, using the sensitivity of the state transitions of the phase trajectories of the system (from the chaotic state to the large-scale periodic state) to the weak signal input, combining with the filter capability of the correlation method.

**Keywords:** weak signal detection, compound system; filter capability; stable operation

### 1 Introduction

In weak signal detection process, the signal input to the detection system is often accompanied by the noise [1, 2]. The word weak in weak signal detection not only refers that the amplitude of signal to be measured is low, and means that the ratio of the measured signal amplitude and noise power is low. That is to say, the purpose of the weak signal detection is to detect weak signal submerged in strong noise. Therefore, the noise is unfavorable factors in the processing of weak signal detection. How to suppress noise to improve effectively the SNR threshold is a difficult problem to be solved in the field of weak signal detection.

In view of the weak signal detection technology, the foreign develops rapidly. In 1942 the establishment of Wiener filtering theory [3-5] greatly promoted the development of the weak signal detection theory in the world, and at the same time, domestic personnel engaged in related research is less. From then on, all kinds of new method of weak signal detection theory emerge in endlessly and a variety of excellent performance testing equipment have been introduced which greatly improves the detection accuracy of weak signal. The detection of weak fault signal using chaotic data sequence analysis and fault diagnosis technology has been up in recent years, and becomes a new and effective method[6]. Therefore, the combination of the chaos theory and weak signal detection is of epoch-making significance, it opens up a new application field of chaos theory, actually provides a new theoretical foundation and support for signal detection technology in the application of engineering.

With the development of weak signal detection, the types of weak signal increases, the requirements for detection accuracy are also getting higher and higher. The method of using a single weak signal detection hasn't gradually meet the detection needs of complex industrial process. However, since the result of detection is strongly influenced by noise and signal to noise ratio threshold of detection is rather high, the method fails to guarantee higher detection accuracy. Therefore, it has become a new hotspot to adopt a combination of a variety of detection methods and gives full play to the advantages of detection means.

This paper presents a compound system for unknown frequency weak signal detection that combines the detection method of double coupling Duffing oscillator and cross-correlation method. This method can fully play their respective advantages and extract periodic signal under test from strong noise that greatly reduces the SNR threshold of the signal.

## 2 Weak signal detection with unknown frequency based on the double coupling doffing oscillator

1) The characteristics of double coupling Duffing system

The specific form of equation for the single Duffing is defined as

$$\ddot{x}(t) + k\dot{x}(t) - x(t) + x^3(t) = F \cos(\omega t) \quad (1)$$

Where,  $x(t)$  is variable of chaotic system,  $k$  is damping ratio,  $t$  is time variable,  $F \cos(\omega t)$  is periodic driving force, where  $F$  is periodic driving force amplitude and  $\omega$  is the angular frequency of the driving cycle.  $-x(t) + x^3(t)$  is the nonlinear restoring force. In the case of  $k$  stationary, the system state changes regularly with the amplitude of driving force  $F \cos(\omega t)$ .

Two coupled Duffing system, a kind of improved Duffing system, is coupled up with the two single Duffing system in order to eliminate some noise through coupling. Its concrete form is defined as

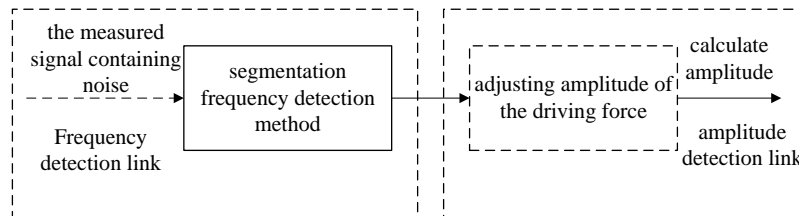
$$\begin{cases} \ddot{x}(t) + k\dot{x}(t) - x(t) + x^3(t) + d(y(t) - x(t)) \\ \quad = F \cos(\omega t) \\ \ddot{y}(t) + k\dot{y}(t) - y(t) + y^3(t) + d(x(t) - y(t)) \\ \quad = F \cos(\omega t) + Y(t) \end{cases} \quad (2)$$

Where,  $x$  is the variable of the first Duffing system,  $y$  is the variable of the second Duffing system,  $d$  is coupling coefficient (in this paper, it is  $d=2$ ), when it is  $d = 0$  coupling of the two systems disappears, dynamic behavior of the system and the single Duffing oscillator system behavior are identical. When it is  $d \neq 0$  variables  $x$ ,  $y$  of the two system rapidly tend to synchronization on the effect of coupling.  $Y(t)$  is external input signal of system including the measured signal  $A_s(t)$  and the noise

signal  $\eta(t)$ ,  $A$  is the amplitude of the measured periodic signal and the measured signal frequency has nothing to do with the driving frequency. Under the condition of fixed damping ratio, phase trajectory of two coupled Duffing system  $F \cos(\omega t)$  changes with the amplitude of driving force.

2) The detection process

A method for detecting weak signal with unknown frequency based on double coupling Duffing oscillator consists of two detection links: frequency detection link and amplitude detection link. The detection process is just like Fig.1, the measured signal containing noisy is input to double coupling Duffing system, then it can detect the frequency of the measured signal through piecewise measuring frequency detection method. In the process of the amplitude detection, firstly the reference frequency of the driving signal is measured as the measured signal. Then the measured signal with noisy is input to double coupling Duffing system, detecting amplitude of the measured signal.



**Fig.1.** The detection process of detecting weak signal method with unknown frequency based on double coupling Duffing oscillator

### 3 Weak Signal Detection Based On Cross-Correlation Method

In the method of weak signal detection, time domain method is the first developed detection method and related method is one of the most commonly used traditional time-domain detection method. The basic idea of weak signal detection based on cross-correlation method is that, in the situation of the measured signal with known frequency, the measured signal mixed with noise and reference signal with the same frequency is made cross-correlation arithmetic, finally the magnitude of the measured signal is determined according to the output of amplitude of correlation function. Weak signal detection based on the autocorrelation method is to deal with the measured signal and itself, regarded as a special case of cross-correlation method.

The input signal mixed with noise is defined as

$$F_1(t) = f_1(t) + n(t) \quad (3)$$

The reference signal with the same frequency as the measured signal is defined as

$$F_2(t) = f_2(t) \quad (4)$$

Where,  $f_1(t)$  is the measured weak periodic signal,  $n(t)$  is noise signal,  $f_2(t)$  is the reference signal with the same frequency with  $f_1(t)$ , but  $n(t)$  and  $f_2(t)$  are irrelevant, the cross-correlation function  $R_{F_1F_2}(\tau)$  is defined as

$$\begin{aligned} R_{F_1F_2}(\tau) &= \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T F_1(t)F_2(t-\tau)dt \\ &= \lim_{T \rightarrow \infty} \left[ \frac{1}{2T} \int_{-T}^T f_1(t)f_2(t-\tau)dt \right. \\ &\quad \left. + \frac{1}{2T} \int_{-T}^T n(t)f_2(t-\tau)dt \right] \\ &= R_{f_1f_2}(\tau) + R_{nf_2}(\tau) \end{aligned} \quad (5)$$

Where,  $R_{f_1f_2}(\tau)$  is the cross-correlation function of  $f_1(t)$  and  $f_2(t)$ ,  $R_{nf_2}(\tau)$  is the cross-correlation function of  $n(t)$  and  $f_2(t)$ , the reference signal  $f_2(t)$  and noise signal  $n(t)$  is uncorrelated in the ideal state, if the mean value of  $n(t)$  or  $f_2(t)$  is zero. The cross-correlation function  $R_{nf_2}(\tau)$  can be considered as zero.

Assumption

$$F_1(t) = f_1(t) + n(t) = A_1 \sin \omega t + n(t) \quad (6)$$

$$F_2(t) = f_2(t) = A_2 \sin \omega t \quad (7)$$

Where,  $A_1$  is amplitude of the measured weak periodic signal  $f_1(t)$ ,  $A_2$  is amplitude of the reference signal  $f_2(t)$ . Then we get

$$R_{F_1F_2}(\tau) = R_{f_1f_2}(\tau) = \frac{A_1 A_2}{2} \cos \omega \tau = A \cos \omega \tau \quad (8)$$

Where,  $A$  is the amplitude of cross-correlation function  $R_{F_1F_2}(\tau)$ .

Thus, in the ideal state, the cross-correlation function calculation is the cosine signals with the same frequency as the measured periodic function. As noise hasn't any impact on periodicity of the signal, the cross-correlation detection method has a certain ability to filter noise signal. Through formula (8) the amplitude of the measured periodic signal can be obtained

$$A_1 = \frac{2A}{A_2} \quad (9)$$

But in fact, the noise signal can't be completely uncorrelated with the reference signal,  $R_{nf_2}(\tau)$  also may not is zero. Therefore, the cross-correlation function output is bound to contain some noise, which greatly limits the use of cross-correlation method to detect the noise ratio of weak signal.

## 4 Conclusion

Through studying the advantage and disadvantage of weak signal detection method based on double coupling Duffing oscillator and cross-correlation detection method, this paper proposes a compound system for weak signal detection with unknown frequency. The compound system is combined with correlation method and double coupling Duffing oscillator, correlation method has the advantage of filtering part of noise signal, double coupling Duffing oscillator has the advantage of testing the measured periodic signal with unknown frequency and extracting the periodic signal submerged in noise signal. Through a series of simulation experiments, it shows that the system can better extract the measured periodic signal from strong noise and can greatly reduce the SNR threshold of the signal. This method not only solves the problem that weak signal detection method based on chaotic system easily generates the misjudgment affected by noise, but also breaks through the limitations that the existing chaotic oscillator detection method can only detect signal with known frequency.

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