

Quality Assessment for Nonlinear Dimensionality Reduction using Procrustes Analysis

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Abstract. In order to achieve of PNL-ICA (Post-Nonlinear Independent Component Analysis) by using Dimensionality Reduction [1] the data set has to be embedded in a hyperplane, i.e. linear combination of the latent variables. This condition must be fulfilled in order to ensure that there is an unique solution to the problem. This article describes a new quality measure for Nonlinear Dimensionality Reduction based on Procrustes Analysis. This approach aims to solve the question of how to evaluate if a low-dimensional embedding (outcome of the process of dimensional reduction) can be used to recover data by using ICA methods.

Keywords: Nonlinear Dimensional Reduction, Post-Nonlinear Independent Component Analysis, Procrustes Analysis

1 Introduction

In the PNL-ICA case, the data set is in a manifold in \mathbf{R}^D , this manifold is in turn in a high-dimensional space with dimension $D > P$, this means that this data set does not fill the space completely. Hence, we can find a lower dimensional space (\mathbf{R}^P) where most of the variance of the data can be explained. It is valid to consider a subset of the data as a geometrical *Shape* with dimension $k \times D$, where k is the number of data points of the subset and D is dimension of the data before dimensional embedding. We can use the fact that every point of this subset is labeled, with preassigned correspondence between the original and the embedded data. Such data sets arise often in a biological or medical setting, when corresponding labeled points are called *landmarks*. Other areas in which landmark data arise include archaeology, astronomy, cartography, manufacturing, geology and geophysics. Thus in some instances landmarks may refer to the same physical markers identifiable in more than one map, satellite image, X-ray, etc.

2 Landmarks, Shape and Euclidean Distance

Suppose we have a post-nonlinear mixture of data \mathbf{y} , with $D \times N$ labeled points which can be embedded in a P -dimensional hyperplane $\hat{\mathbf{z}}$, this two sets- \mathbf{y} and $\hat{\mathbf{z}}$ - are closely related for two main reasons. First, because every data label in the P -dimensional space correspond to the same data point in the D -dimensional embedding (one-to-one relationship). Second, the information conveyed in the first set has to be coincident with the information conveyed in the second set. Since the model of PNL-ICA is isometric, then the key idea to obtain a satisfactory result is maintain the pair-wise geodesic distance when the data set is embedded into the low-dimensional space, if this part of the process is accurate, then the outcome will reduce the problem to a linear mixture version[1].

We run the experiment explained above using 4 NLDR algorithms, which were extracted from *MANifold Learning Matlab Demo* in <http://www.math.ucla.edu/~wittman/mani/>. We compared the goodness of fit evaluated with Trustworthiness, Continuity [3] and Geodesic Dissimilarity, the results are shown in Table 1. In this table, we can clearly see that Trustworthiness is a quite insensitive to evaluate the task of interest. Geodesic Dissimilarity can be used to compare the performance with different NLDR algorithms in order to choose the embedding that is most effective, the one with the lowest Geodesic Dissimilarity.

Algorithm	Trustworthiness	Continuity	Geodesic Dissimilarity	Is effective?
Locally Linear Embedding[4]	0.9991	0.9991	0.0710	YES
Curvilinear Component Analysis[5]	0.9765	0.9781	0.3512	NO
ISOMAP[6]	0.9999	0.9999	0.0013	YES
Laplacian Eigenmaps[7]	0.9972	0.9978	0.1754	NO

Table 1. Trustworthiness, Continuity and Geodesic Dissimilarity for the experiment described in the Section 4.

5 CONCLUSIONS

The disadvantage of the measures that are based on all pairwise distances is that they are not directly related to any specific task. Geodesic Dissimilarity offers a specific quality measure for dimensionality algorithms that embed the nonlinear manifold into a hyperplane and is directly related to the task where the user is trying to separate latent variables using ICA. This approach has been tested with speech signals with good results, with several experiments we can suggest that an embedding with Geodesic Dissimilarity less than 0.1 is effective to be used with ICA methods. An interesting fact is that even non isometric NLDR algorithms also perform well for PNL-ICA, a deeper study of this phenomenon is left as future work. An important advantage of this quality measure is that it is not directly related to visualization, it is more related to the linear correlation between the data in the two embeddings, which can be useful in a number of applications, not only in PNL-ICA.

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