

# Undecimated Non-uniform Multivariate Empirical Mode Decomposition Filter banks for Arbitrary Nodes and its Application for Speech Enhancement

Min-sung Koh

School of Computing and Engineering Sciences  
Eastern Washington University,  
Cheney WA 99004, USA  
[mkoh@ewu.edu](mailto:mkoh@ewu.edu)

**Abstract:** This paper introduces a technique to build *undecimated* Multivariate Empirical Mode Decomposition Filter Banks (MEMDFBs) for *arbitrary trees*. The available option of undecimated MEMDFBs for arbitrary trees is achieved by exchanging assisting noise channels. An application for speech enhancement is also introduced.

## 1 Introduction

Empirical mode decomposition (EMD) introduced in [1] is an efficient signal processing technique for non-linear and non-stationary signals. The EMD was extended to multivariate EMD (MEMD) in [2] for multichannel applications such as multichannel speech signals, EEG signals etc. Recently, a theory to build non-uniform filter banks using the MEMD was also introduced in [3]. The paper [3] shows how to build decimated/undecimated MEMD filter banks (MEMDFBs) for any arbitrary trees. However, *undecimated* MEMDFBs are applicable *only* to octave trees because many node signals do not exist for other trees, whereas *decimated* MEMDFBs are available for any arbitrary tree structures [3]. This paper resolves the issue in order to make *undecimated* MEMDFBs available for arbitrary trees, which are essential for various non-uniform filter banks.

## 2 Undecimated and Exchanged-NA-MEMDFBs

The MEMD in [2] projects the given signals onto uniformly spaced direction vectors in a hypersphere to extract intrinsic mode functions (IMFs) and the residual [2]. The MEMD produces IMFs for each one of the channels in the given multivariate signal. To extend the MEMD to MEMDFBs, only the 1<sup>st</sup> IMF and one residual are considered [3]. A MATLAB-like pseudo code for the MEMD to be used in MEMDFBs is given in Fig.1. The algorithm in Fig. 1 is basically identical to the MEMD in [2, 5] except for the fact that only the 1<sup>st</sup> IMF and one residual are obtained.

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function [L, H] = OneIMFOneResidualMEMD(X,Nd)
1. [L Nc] = size(X); %Nc is the number of channels.
2. Using Hammersley sequence, choose pointset, S, on a hypersphere for direction vectors,
   where S = (Nc x Nd) and Nd is the number of direction vectors.
3. PotentialIMF = X;
4. E(t) = averaged envelope from “b” to “f” in step 5;
5. While (sifting stop condition is not satisfied)
   a. PotentialIMF = PotentialIMF – E(t);
   b. Make direction vectors, dk, using Hammersley seq, where dk = (1 x Nc) and k
      =1,2, ... Nd.
   c. Project multivariate signals onto the direction vectors. (i.e, pk(t) = dk*PotentialIMF
      T;)
   d. Obtain local maxima and minima using pk(t), where the projected signal, pk(t), is (1
      x L).
   e. Using the time values of local maxima /minima and the max/min values of each
      channel, interpolate local maxima and minima to obtain upper, lower, and mean en-
      velopes for kth direction vector, where kth each envelop is (L x Nc).
   f. Obtain the averaged envelope, E(t), by averaging mean envelopes for all direction
      vectors, k=1,2,..., Nd ( i.e., E(t) = sum of all mean envelopes / Nd , where E(t)
      has (L x Nc) data.)
   end
6. H = PotentialIMF;
7. L = X – H;

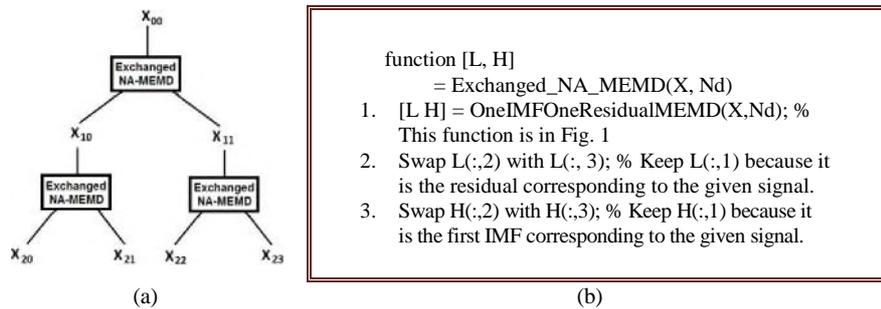
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Fig. 1. Pseudo code for the “MEMD” to obtain only the 1<sup>st</sup> IMF and one residual.

In EMD and MEMD, signals without enough number of local minima and/or maxima (e.g., an impulse signal) are *not* successfully decomposed. To overcome the problem, ensemble EMD (EEMD) was introduced in [4], where a small zero-mean white Gaussian noise is directly added into a given signal to provide sufficient number of local maxima and minima. Then, each IMF is ensemble averaged to remove added noise portion buried in the IMF. Although EEMD resolves the problem, it has a drawback in reconstructing original signal if enough ensemble mean is not applied [3, 5]. Similar principle was introduced in [5] for MEMD and it was coined noise aided MEMD (NA-MEMD), where, however, the noises are *not* directly added into a given multichannel signal. The NA-MEMD puts noises in a few separate channels of MEMD. Since the MEMD [2] and NA-MEMD [5] project multivariate signals onto multi-dimensional *fixed* direction vectors, an idea of changing direction vectors is introduced in this paper so that *undecimated* NA-MEMDFBs in [3] is extended to any *arbitrary* trees. Instead of explicit changing of direction vectors, an idea to exchange aiding noise channels is adopted. Exchanging aiding noise channels causes a relative change of direction vectors for the projecting signal in NA-MEMDFBs. As the result, projection can be obtained at different directions. The proposed algorithm, coined “Undecimated and exchanged-NA-MEMDFBs”, uses basically the same idea with NA-MEMDFBs in [3] but it is different in exchanging noise channels for a next decomposition.

A tree structure for MEMDFBs is given in Fig. 2(a), where the algorithm denoted by “Exchanged-NA-MEMD” is given in Fig. 2 (b) as a pseudo code. In Fig. 2(b), the

input  $X$  includes assisting noise channels in separate column vectors. Fig. 2(b) assumes that the given signal,  $X$ , has a single channel signal and two aiding noise channels. If the original signal has multichannel, then two assisting noise channels are added into next two column vectors after multiple signal vectors. In Fig. 2(b), the assisting two noise channels are generated by zero-mean white Gaussian noises with 20 [dB] in SNR. The  $H$  and  $L$  in Fig. 2(b) are respectively corresponding to the first IMF and the residual at a node. Notice that perfect reconstruction is also achieved in the proposed algorithm because aiding noise channels are *not directly* added into original signals.



**Fig. 2.** Proposed algorithm, which is undecimated and exchanged NA-MEMDFBs. (a): a full binary tree at decomposition level of 2. (b): Pseudo code for the “Exchanged NA-MEMD” in (a).

### 3 Experimental Results

To test the proposed algorithm in this paper, a special tree structure to approximate critical bands is considered even though it can be applied to any arbitrary tree structures. For speech signals, a particular non-uniform filter banks approximating critical bands of human auditory system in [6] is built through the tree structure given in [6]. All 25 end-node signals of the tree in [6] are applied to the speech enhancement introduced in [7] for 10 continuous speech files. Those results are shown in Table 1, where the columns denoted by “w/o exchanging” imply that the undecimated MEMDFBs in [3] are applied with the same speech enhancement algorithm for comparison. Through Table 1 and informal listening tests, it is observed that the proposed algorithm makes *less residual noise* and works better in low SNRs for non-white noises (i.e., pink and babble noises) than the undecimated MEMDFBs in [3]. In addition, it is observed that the proposed algorithm works better in noise-dominant frames. For white noises and high SNRs, the proposed algorithm and the undecimated MEMDFBs in [3] show a similar performance.

Input SNRs [dB]	White noise [dB]		Pink noise [dB]		Babble noise [dB]	
	w/o exchanging	Proposed	w/o exchanging	Proposed	w/o exchanging	Proposed
-1	9.0044	<b>9.1061</b>	2.4298	<b>4.3392</b>	1.8508	<b>2.8454</b>

1	10.2396	<b>10.3752</b>	4.2138	<b>5.5372</b>	3.5838	<b>4.4471</b>
3	11.5065	<b>11.6314</b>	5.9308	<b>6.8049</b>	5.4222	<b>5.9250</b>
5	12.7920	<b>12.9158</b>	7.7111	<b>8.2094</b>	7.1386	<b>7.5361</b>
7	14.1347	<b>14.2455</b>	9.4779	<b>9.6377</b>	8.9538	<b>9.2197</b>
9	15.5444	<b>15.6090</b>	11.2579	11.2280	10.7581	<b>10.9479</b>
11	17.0201	17.0125	13.0737	12.9005	12.6007	<b>12.7720</b>

Table 1. SNR comparison between the proposed algorithm and the undecimated MEMDFBs [3] without exchanging for the same speech enhancement algorithm.

## 4 Conclusions

In this paper, *undecimated and exchanged*-NA-MEMDFBs are developed by an idea to change direction vectors for the projection at each node. Changing the direction vectors is achieved through exchanging of aiding noise channels in NA-MEMDFBs. Since noise channels in NA-MEMDFBs are just assisting MEMD decomposition, the exchanging of noise channels in this paper does not affect perfect reconstruction given in [3]. Undecimated NA-MEMDFBs in [3] are available *only* to octave trees, whereas the proposed algorithm is applicable to *any* arbitrary tree structures for diverse nonuniform filter banks.

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